

# 2021 AMC 10A (November) Problems and Answer Key \*

PROFESSOR CHEN EDUCATION PALACE

1. (2021 AMC 10A (Nov) Problem 1)(2021 AMC 12A (Nov) Problem 1)

What is the value of  $\frac{(2112-2021)^2}{169}$ ?

- A. 7   B. 21   C. 49   D. 64   E. 91

**Solution:** (C)

2. (2021 AMC 10A (Nov) Problem 2)(2021 AMC 12A (Nov) Problem 2)

Menkara has a  $4 \times 6$  index card. If she shortens the length of one side of this card by 1 inch, the card would have areas 18 square inches. What would the area of the card be in square inches if instead she shortens the length of the other side by 1 inch?

- A. 16   B. 17   C. 18   D. 19   E. 20

**Solution:** (E)

3. (2021 AMC 10A (Nov) Problem 3)

What is the maximum number of balls of clay with radius 2 that can completely fit inside a cube of side length 6 assuming that the balls can be reshaped but not compressed before they are packed in the cube?

- A. 3   B. 4   C. 5   D. 6   E. 7

**Solution:** (D)

4. (2021 AMC 10A (Nov) Problem 4)(2021 AMC 12A (Nov) Problem 3)

Mr. Lopez has a choice of two routes to get to work. Route *A* is 6 miles long, and his average speed along this route is 30 miles per hour. Route *B* is 5 miles long, and his average speed

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along this route is 40 miles per hour, except for a  $\frac{1}{2}$ -mile stretch in a school zone where his average speed is 20 miles per hour. By how many minutes is Route  $B$  quicker than Route  $A$ ?  
A.  $2\frac{3}{4}$  B.  $3\frac{3}{4}$  C.  $4\frac{1}{2}$  D.  $5\frac{1}{2}$  E.  $6\frac{3}{4}$

**Solution:** (B)

5. (2021 AMC 10A (Nov) Problem 5)(2021 AMC 12A (Nov) Problem 4)

The six-digit number  $\underline{20210A}$  is prime for only one digit  $A$ . What is  $A$ ?

A. 1 B. 3 C. 5 D. 7 E. 9

**Solution:** (E)

6. (2021 AMC 10A (Nov) Problem 6)(2021 AMC 12A (Nov) Problem 5)

Elmer the emu takes 44 equal strides to walk between consecutive telephone poles on a rural road. Oscar the ostrich can cover the same distance in 12 equal leaps. The telephone poles are evenly spaced, and the 41st pole along this road is exactly one mile (5280 feet) from the first pole. How much longer, in feet, is Oscar's leap than Elmer's stride?

A. 6 B. 8 C. 10 D. 11 E. 15

**Solution:** (B)

7. (2021 AMC 10A (Nov) Problem 7)(2021 AMC 12A (Nov) Problem 6)

As shown in the figure below, point  $E$  lies in the opposite half-plane determined by line  $CD$  from point  $A$  so that  $\angle CDE = 110^\circ$ . Point  $F$  lies on  $\overline{AD}$  so that  $DE = DF$ , and  $ABCD$  is a square. What is the degree measure of  $\angle AFE$ ?

A. 160 B. 164 C. 166 D. 170 E. 174

**Solution:** (D)

8. (2021 AMC 10A (Nov) Problem 8)

A two-digit positive integer is said to be *cuddly* if it is equal to the sum of its nonzero tens digit and the square of its units digit. How many two-digit positive integers are cuddly?

A. 0 B. 1 C. 2 D. 3 E. 4

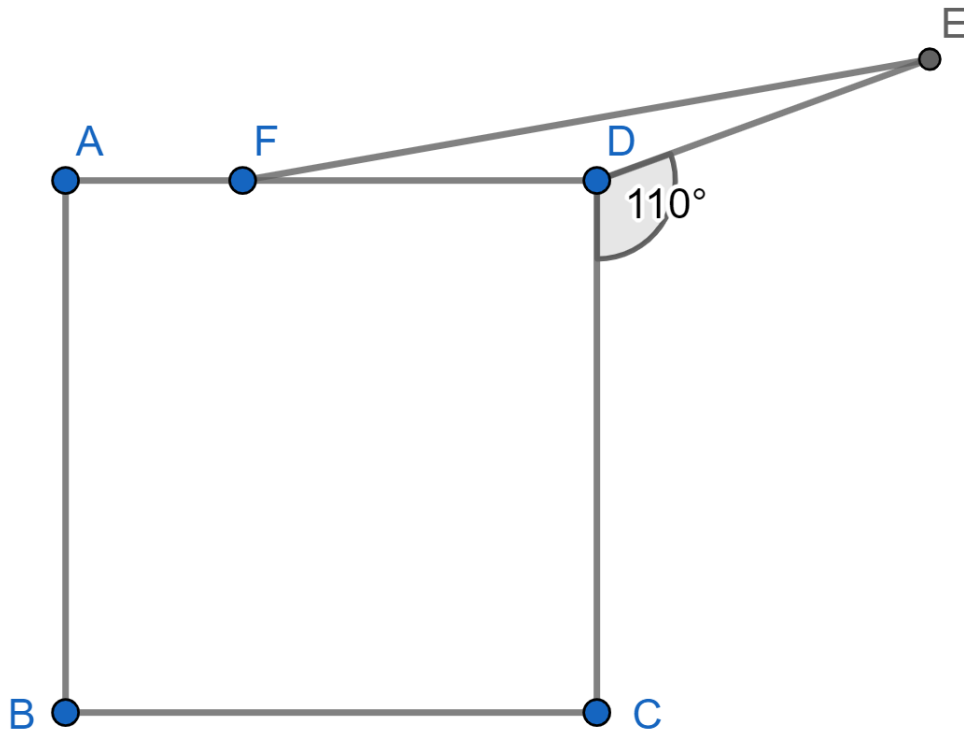


Figure 1: 2021 AMC 10A (Nov) Problem 7

Solution: (B)

9. (2021 AMC 10A (Nov) Problem 9)

When a certain unfair die is rolled, an even number is 3 times as likely to appear as an odd number. The die is rolled twice. What is the probability that the sum of the numbers rolled is even?

- A.  $\frac{3}{8}$  B.  $\frac{4}{9}$  C.  $\frac{5}{9}$  D.  $\frac{9}{16}$  E.  $\frac{5}{8}$

Solution: (E)

10. (2021 AMC 10A (Nov) Problem 10)(2021 AMC 12A (Nov) Problem 7)

A school has 100 students and 5 teachers. In the first period, each student is taking one class, and each teacher is teaching one class. The enrollments in the classes are 50, 20, 20, 5, and 5. Let  $t$  be the average value obtained if a teacher is picked at random and the number of students

in their class is noted. Let  $s$  be the average value obtained if a student is picked at random and the number of students in their class, including that student, is noted. What is  $t - s$ ?

- A. -18.5   B. -13.5   C. 0   D. 13.5   E. 18.5

**Solution:** (B)

11. (2021 AMC 10A (Nov) Problem 11)

Emily sees a ship traveling at a constant speed along a straight section of a river. She walks parallel to the riverbank at a uniform rate faster than the ship. She counts 210 equal steps walking from the back of the ship to the front. Walking in the opposite direction, she counts 42 steps of the same size from the front of the ship to the back. In terms of Emily's equal steps, what is the length of the ship?

- A. 70   B. 84   C. 98   D. 105   E. 126

**Solution:** (A)

12. (2021 AMC 10A (Nov) Problem 12)(2021 AMC 12A (Nov) Problem 10)

The base-nine representation of the number  $N$  is  $27,006,000,052_{\text{nine}}$ . What is the remainder when  $N$  is divided by 5?

- A. 0   B. 1   C. 2   D. 3   E. 4

**Solution:** (D)

13. (2021 AMC 10A (Nov) Problem 13)

Each of 6 balls is randomly and independently painted either black or white with equal probability. What is the probability that every ball is different in color from more than half of the other 5 balls?

- A.  $\frac{1}{64}$    B.  $\frac{1}{6}$    C.  $\frac{1}{4}$    D.  $\frac{5}{16}$    E.  $\frac{1}{2}$

**Solution:** (D)

14. (2021 AMC 10A (Nov) Problem 14)

How many ordered pairs  $(x, y)$  of real numbers satisfy the following system of equations?

$$x^2 + 3y = 9$$

$$(|x| + |y| - 4)^2 = 1$$

- A. 1   B. 2   C. 3   D. 5   E. 7

**Solution:** (D)

15. (2021 AMC 10A (Nov) Problem 15)

Isosceles triangle  $ABC$  has  $AB = AC = 3\sqrt{6}$ , and a circle with radius  $5\sqrt{2}$  is tangent to line  $AB$  at  $B$  and to line  $AC$  at  $C$ . What is the area of the circle that passes through vertices  $A$ ,  $B$ , and  $C$ ?

- A.  $24\pi$    B.  $25\pi$    C.  $26\pi$    D.  $27\pi$    E.  $28\pi$

**Solution:** (C)

16. (2021 AMC 10A (Nov) Problem 16)

The graph of

$$f(x) = \lfloor x \rfloor - \lfloor 1 - x \rfloor$$

is symmetric about which of the following? (Here  $\lfloor x \rfloor$  is the greatest integer not exceeding  $x$ .)

- A. the  $y$ -axis   B. the line  $x = 1$    C. the origin   D. the point  $(\frac{1}{2}, 0)$    E. the point  $(1, 0)$

**Solution:** (D)

17. (2021 AMC 10A (Nov) Problem 17)

An architect is building a structure that will place vertical pillars at the vertices of regular hexagon  $ABCDEF$ , which is lying horizontally on the ground. The six pillars will hold up a flat solar panel that will not be parallel to the ground. The heights of the pillars at  $A$ ,  $B$ , and  $C$  are 12, 9, and 10 meters, respectively. What is the height, in meters, of the pillar at  $E$ ?

- A. 9   B.  $6\sqrt{3}$    C.  $8\sqrt{3}$    D. 17   E.  $12\sqrt{3}$

**Solution:** (D)

18. (2021 AMC 10A (Nov) Problem 18)

A farmer's rectangular field is partitioned into a 2 by 2 grid of 4 rectangular sections as shown in the figure. In each section the farmer will plant one crop: corn, wheat, soybeans, or potatoes. The farmer does not want to grow corn and wheat in any two sections that share a border,



**Figure 2:** 2021 AMC 10A (Nov) Problem 18

and the farmer does not want to grow soybeans and potatoes in any two sections that share a border. Given these restrictions, in how many ways can the farmer choose crops to plant in each of the four sections of the field?

- A. 12   B. 64   C. 84   D. 90   E. 144

**Solution:** (C)

19. (2021 AMC 10A (Nov) Problem 19)

A disk of radius 1 rolls all the way around the inside of a square of side length  $s > 4$  and sweeps out a region of area  $A$ . A second disk of radius 1 rolls all the way around the outside of the same square and sweeps out a region of area  $2A$ . The value of  $s$  can be written as  $a + \frac{b\pi}{c}$ , where  $a$ ,  $b$ , and  $c$  are positive integers and  $b$  and  $c$  are relatively prime. What is  $a + b + c$ ?

- A. 10   B. 11   C. 12   D. 13   E. 14

**Solution:** (A)

20. (2021 AMC 10A (Nov) Problem 20)(2021 AMC 12A (Nov) Problem 17)

For how many ordered pairs  $(b, c)$  of positive integers does neither  $x^2 + bx + c = 0$  nor  $x^2 + cx + b = 0$  has two distinct real solutions?

A. 4   B. 6   C. 8   D. 12   E. 16

**Solution:** (B)

21. (2021 AMC 10A (Nov) Problem 21)(2021 AMC 12A (Nov) Problem 18)

Each of 20 balls is tossed independently and at random into one of 5 bins. Let  $p$  be the probability that some bin ends up with 3 balls, another with 5 balls, and the other three with 4 balls each. Let  $q$  be the probability that every bin ends up with 4 balls. What is  $\frac{p}{q}$ ?

A. 1   B. 4   C. 8   D. 12   E. 16

**Solution:** (E)

22. (2021 AMC 10A (Nov) Problem 22)

Inside a right circular cone with base radius 5 and height 12 are three congruent spheres each with radius  $r$ . Each sphere is tangent to the other two spheres and also tangent to the base and side of the cone. What is  $r$ ?

A.  $\frac{3}{2}$    B.  $\frac{90-40\sqrt{3}}{11}$    C. 2   D.  $\frac{144-25\sqrt{3}}{44}$    E.  $\frac{5}{2}$

**Solution:** (B)

23. (2021 AMC 10A (Nov) Problem 23)(2021 AMC 12A (Nov) Problem 20)

For each positive integer  $n$ , let  $f_1(n)$  be twice the number of positive integer divisors of  $n$ , and for  $j \geq 2$ , let  $f_j(n) = f_1(f_{j-1}(n))$ . For how many values of  $n \leq 50$  is  $f_{50}(n) = 12$ ?

A. 7   B. 8   C. 9   D. 10   E. 11

**Solution:** (D)

24. (2021 AMC 10A (Nov) Problem 24)

Each of the 12 edges of a cube is labeled 0 or 1. Two labelings are considered different even if one can be obtained from the other by a sequence of one or more rotations and/or reflections. For how many such labelings is the sum of the labels on the edges of each of the 6 faces of the cube equal to 2?

- A. 8   B. 10   C. 12   D. 16   E. 20

**Solution:** (E)

25. (2021 AMC 10A (Nov) Problem 25)(2021 AMC 12A (Nov) Problem 23)

A quadratic polynomial  $p(x)$  with real coefficients and leading coefficient 1 is called *disrespectful* if the equation  $p(p(x)) = 0$  is satisfied by exactly three real numbers. Among all the disrespectful quadratic polynomials, there is a unique such polynomial  $\tilde{p}(x)$  for which the sum of the roots is maximized. What is  $\tilde{p}(1)$ ?

- A.  $\frac{5}{16}$    B.  $\frac{1}{2}$    C.  $\frac{5}{8}$    D. 1   E.  $\frac{9}{8}$

**Solution:** (A)