2022 AMC 10A
Problems and Answer Key *

PROFESSOR CHEN EDUCATION PALACE

1. (2022 AMC 10A Problem 1)(2022 AMC 12A Problem 1)
   What is the value of
   $$3+\frac{1}{3+\frac{1}{3+\frac{1}{3}}}?$$
   A. $\frac{31}{10}$ B. $\frac{49}{15}$ C. $\frac{33}{10}$ D. $\frac{109}{33}$ E. $\frac{15}{4}$

   Solution: **(D)**

2. (2022 AMC 10A Problem 2)
   Mike cycled 15 laps in 57 minutes. Assume he cycled at a constant speed throughout. Approximately how many laps did he complete in the first 27 minutes?
   A. 5 B. 7 C. 9 D. 11 E. 13

   Solution: **(B) 7**

3. (2022 AMC 10A Problem 3)(2022 AMC 12A Problem 2)
   The sum of three numbers is 96. The first number is 6 times the third number, and the third number is 40 less than the second number. What is the absolute value of the difference between the first and second numbers?
   A. 1 B. 2 C. 3 D. 4 E. 5

   Solution: **(E)**

4. (2022 AMC 10A Problem 4)

   In some countries, automobile fuel efficiency is measured in liters per 100 kilometers while other countries use miles per gallon. Suppose that 1 kilometer equals m miles, and 1 gallon equals l liters. Which of the following gives the fuel efficiency in liters per 100 kilometers for a car that gets x miles per gallon?

   A. $\frac{x}{1000ml}$ B. $\frac{xlm}{100}$ C. $\frac{lm}{100x}$ D. $\frac{100}{xlm}$ E. $\frac{100ml}{x}$

Solution: **(E)** $\frac{100ml}{x}$

5. (2022 AMC 10A Problem 5)

   Square $ABCD$ has side length 1. Points P, Q, R, and S each lie on a side of $ABCD$ so that $APQCRS$ is an equilateral convex hexagon with side length s. What is s?

   A. $\frac{\sqrt{2}}{3}$ B. $\frac{1}{2}$ C. $2-\sqrt{2}$ D. $1-\frac{\sqrt{2}}{4}$ E. $\frac{2}{3}$

Solution: **(C)** $2-\sqrt{2}$

6. (2022 AMC 10A Problem 6)

   Which expression is equal to $\left|a-2-\sqrt{(a-1)^2}\right|$ for $a<0$?

   A. $3-2a$ B. $1-a$ C. 1 D. $a+1$ E. 3

Solution: **(A)** $3-2a$

7. (2022 AMC 10A Problem 7)(2022 AMC 12A Problem 4)

   The least common multiple of a positive integer n and 18 is 180, and the greatest common divisor of n and 45 is 15. What is the sum of the digits of n?

   A. 3 B. 6 C. 8 D. 9 E. 12

Solution: **(B)**

8. (2022 AMC 10A Problem 8)(2022 AMC 12A Problem 6)

   A data set consists of 6 (not distinct) positive integers: 1, 7, 5, 2, 5, and X. The average (arithmetic mean) of the 6 numbers equals a value in the data set. What is the sum of all possible values of X?

   A. 10 B. 26 C. 32 D. 36 E. 40

Solution: **(D)**

9. (2022 AMC 10A Problem 9)(2022 AMC 12A Problem 7)

A rectangle is partitioned into 5 regions as shown. Each region is to be painted a solid color—red, orange, yellow, blue, or green—so that regions that touch are painted different colors, and colors can be used more than once. How many different colorings are possible?

Figure 1: 2022 AMC 12A Problem 7

A. 120 B. 270 C. 360 D. 540 E. 720

Solution: **(D)**

10. (2022 AMC 10A Problem 10)

Daniel finds a rectangular index card and measures its diagonal to be 8 centimeters. Daniel then cuts out equal squares of side 1 cm at two opposite corners of the index card and measures the distance between the two closest vertices of these squares to be $4\sqrt{2}$ centimeters, as shown below. What is the area of the original index card?

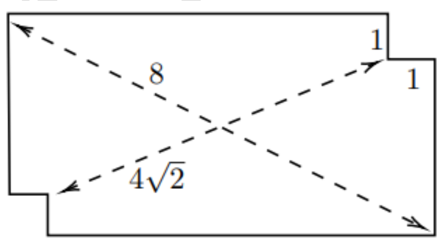

Figure 2: 2022 AMC 10A Problem 10

A. 14 B. $10\sqrt{2}$ C. 16 D. $12\sqrt{2}$ E. 18

Solution: **(E) 18**

11. (2022 AMC 10A Problem 11)

Ted mistakenly wrote $2^m \cdot \sqrt{\frac{1}{4096}}$ as $2 \cdot \sqrt[m]{\frac{1}{4096}}$. What is the sum of all real numbers m for which these two expressions have the same value?

A. 5 B. 6 C. 7 D. 8 E. 9

Solution: **(C) 7**

12. (2022 AMC 10A Problem 12)(2022 AMC 12A Problem 9)

On Halloween 31 children walked into the principal's office asking for candy. They can be classified into three types: Some always lie; some always tell the truth; and some alternately lie and tell the truth. The alternaters arbitrarily choose their first response, either a lie or the truth, but each subsequent statement has the opposite truth value from its predecessor. The principal asked everyone the same three questions in this order.

"Are you a truth-teller?" The principal gave a piece of candy to each of the 22 children who answered yes.

"Are you an alternater?" The principal gave a piece of candy to each of the 15 children who answered yes.

"Are you a liar?" The principal gave a piece of candy to each of the 9 children who answered yes. How many pieces of candy in all did the principal give to the children who always tell the truth?

A. 7 B. 12 C. 21 D. 27 E. 31

Solution: **(A)**

13. (2022 AMC 10A Problem 13)

Let $\triangle ABC$ be a scalene triangle. Point P lies on $\overline{BC}$ so that $\overline{AP}$ bisects $\angle BAC$. The line through B perpendicular to $\overline{AP}$ intersects the line through A parallel to $\overline{BC}$ at point D. Suppose $BP = 2$ and $PC = 3$. What is AD?

A. 8 B. 9 C. 10 D. 11 E. 12

Solution: **(C) 10**

14. (2022 AMC 10A Problem 14)(2022 AMC 12A Problem 10)

How many ways are there to split the integers 1 through 14 into 7 pairs so that in each pair the greater number is at least 2 times the lesser number?

A. 108 B. 120 C. 126 D. 132 E. 144

Solution: **(E)**

15. (2022 AMC 10A Problem 15)

Quadrilateral $ABCD$ with side lengths $AB = 7$, $BC = 24$, $CD = 20$, and $DA = 15$ is inscribed in a circle. The area interior to the circle but exterior to the quadrilateral can be written in the form $\frac{a\pi - b}{c}$, where a, b, and c are positive integers such that a and c have no common prime factor. What is $a + b + c$?

A. 260 B. 855 C. 1235 D. 1565 E. 1997

Solution: **(D) 1565**

16. (2022 AMC 10A Problem 16)(2022 AMC 12A Problem 15)

The roots of the polynomial $10x^3 - 39x^2 + 29x - 6$ are the height, length, and width of a rectangular box (right rectangular prism). A new rectangular box is formed by lengthening each edge of the original box by 2 units. What is the volume of the new box?

A. $\frac{24}{5}$ B. $\frac{42}{5}$ C. $\frac{81}{5}$ D. 30 E. 48

Solution: **(D)**

17. (2022 AMC 10A Problem 17)

How many three-digit positive integers $\underline{abc}$ are there whose nonzero digits a, b, and c satisfy

$$0.\overline{abc} = \frac{1}{3}\left(0.\bar{a} + 0.\bar{b} + 0.\bar{c}\right)?$$

(The bar indicates digit repetition; thus $0.\overline{abc}$ is the infinite repeating decimal $0.\underline{abcabc}\cdots$)

A. 9 B. 10 C. 11 D. 13 E. 14

Solution: **(D) 13**

18. (2022 AMC 10A Problem 18)(2022 AMC 12A Problem 18)

Let T_k be the transformation of the coordinate plane that first rotates the plane k degrees counter-clockwise around the origin and then reflects the plane across the y-axis. What is the least positive integer n such that performing the sequence of transformations $T_1, T_2, T_3, \cdots, T_n$ returns the point $(1, 0)$ back to itself?

A. 359 B. 360 C. 719 D. 720 E. 721

Solution: **(A)**

19. (2022 AMC 10A Problem 19)

Let L_n denote the least common multiple of the numbers $1, 2, 3, \cdots, n$, and let h be the unique positive integer such that

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{17}=\frac{h}{L_{17}}.$$

What is the remainder when h is divided by 17?

A. 1 B. 3 C. 5 D. 7 E. 9

Solution: **(C) 5**

20. (2022 AMC 10A Problem 20)

A four-term sequence is formed by adding each term of a four-term arithmetic sequence of positive integers to the corresponding term of a four-term geometric sequence of positive integers. The first three terms of the resulting four-term sequence are 57, 60, and 91. What is the fourth term of this sequence?

A. 190 B. 194 C. 198 D. 202 E. 206

Solution: **(E) 206**

21. (2022 AMC 10A Problem 21)

A bowl is formed by attaching four regular hexagons of side 1 to a square of side 1. The edges of adjacent hexagons coincide, as shown in the figure. What is the area of the octagon obtained by joining the top eight vertices of the four hexagons, situated on the rim of the bowl?

A. 6 B. 7 C. $5+2\sqrt{2}$ D. 8 E. 9

Solution: **(B) 7**

22. (2022 AMC 10A Problem 22)(2022 AMC 12A Problem 19)

Suppose that 13 cards numbered 1, 2, 3, . . . , 13 are arranged in a row. The task is to pick them up in numerically increasing order, working repeatedly from left to right. In the example below, cards 1, 2, 3 are picked up on the first pass, 4 and 5 on the second pass, 6 on the third pass, 7, 8, 9, 10 on the fourth pass, and 11, 12, 13 on the fifth pass.

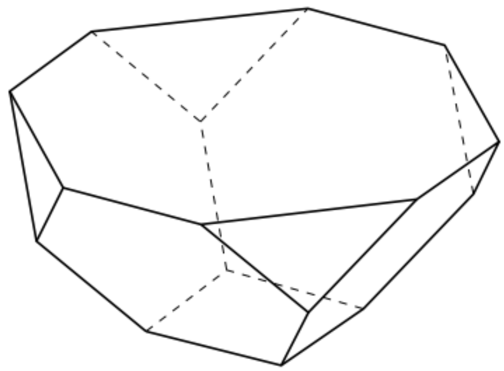

Figure 3: 2022 AMC 10A Problem 21

7	**11**	**8**	**6**	**4**	**5**	**9**	**12**	**1**	**13**	**10**	**2**	**3**

Figure 4: 2022 AMC 12A Problem 19

For how many of the 13! possible orderings of the cards will the 13 cards be picked up in exactly two passes?

A. 4082 B. 4095 C. 4096 D. 8178 E. 8191

Solution: **(D)**

23. (2022 AMC 10A Problem 23)(2022 AMC 12A Problem 20)

Isosceles trapezoid $ABCD$ has parallel sides $\overline{AD}$ and $\overline{BC}$, with $BC < AD$ and $AB = CD$. There is a point P in the plane such that $PA = 1$, $PB = 2$, $PC = 3$, and $PD = 4$. What is $\frac{BC}{AD}$?

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. $\frac{3}{4}$

Solution: **(B)**

24. (2022 AMC 10A Problem 24)(2022 AMC 12A Problem 24)

How many strings of length 5 formed from the digits 0, 1, 2, 3, 4 are there such that for each $j \in \{1, 2, 3, 4\}$, at least j of the digits are less than j? (For example, 02214 satisfies this condition because it contains at least 1 digit less than 1, at least 2 digits less than 2, at least 3 digits less than 3, and at least 4 digits less than 4. The string 23404 does not satisfy the condition because it does not contain at least 2 digits less than 2.)

A. 500 B. 625 C. 1089 D. 1199 E. 1296

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Solution: **(E)**

25. (2022 AMC 10A Problem 25)

Let R, S, and T be squares that have vertices at lattice points (i.e., points whose coordinates are both integers) in the coordinate plane, together with their interiors. The bottom edge of each square is on the x-axis. The left edge of R and the right edge of S are on the y-axis, and R contains $\frac{9}{4}$ as many lattice points as does S. The top two vertices of T are in $R \cup S$, and T contains $\frac{1}{4}$ of the lattice points contained in $R \cup S$. See the figure (not drawn to scale).

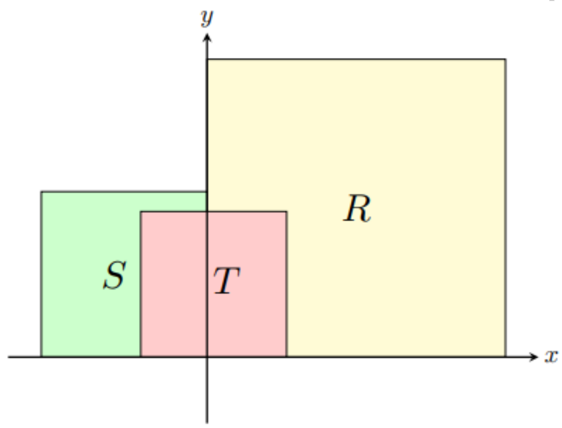

Figure 5: 2022 AMC 10A Problem 25

The fraction of lattice points in S that are in $S \cap T$ is 27 times the fraction of lattice points in R that are in $R \cap T$. What is the minimum possible value of the edge length of R plus the edge length of S plus the edge length of T?

A. 336 B. 337 C. 338 D. 339 E. 340

Solution: **(B) 337**