

2024 AIME II

Problems and Answer Key *

PROFESSOR CHEN EDUCATION PALACE

1. (2024 AIME II Problem 1)

Among the 900 residents of Aimeville, there are 195 who own a diamond ring, 367 who own a set of golf clubs, and 562 who own a garden spade. In addition, each of the 900 residents owns a bag of candy hearts. There are 437 residents who own exactly two of these things, and 234 residents who own exactly three of these things. Find the number of residents of Aimeville who own all four of these things.

Solution: .

2. (2024 AIME II Problem 2)

A list of positive integers has the following properties:

- The sum of the items in the list is 30.
- The unique mode of the list is 9.
- The median of the list is a positive integer that does not appear in the list itself.

Find the sum of the squares of all the items in the list.

Solution: .

3. (2024 AIME II Problem 3)

Find the number of ways to place a digit in each cell of a 2×3 grid so that the sum of the two numbers formed by reading left to right is 999, and the sum of the three numbers formed by reading top to bottom is 99. The grid below is an example of such an arrangement because $8 + 991 = 999$ and $9 + 9 + 81 = 99$.

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0	0	8
9	9	1

Figure 1: 2024 AIME II Problem 3

Solution:

(45).

4. (2024 AIME II Problem 4)

$$\begin{aligned}\log_2 \left(\frac{x}{yz} \right) &= \frac{1}{2} \\ \log_2 \left(\frac{y}{xz} \right) &= \frac{1}{3} \\ \log_2 \left(\frac{z}{xy} \right) &= \frac{1}{4}\end{aligned}$$

Then the value of $|\log_2(x^4y^3z^2)|$ is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Solution: (33).

5. (2024 AIME II Problem 5)

Let $ABCDEF$ be a convex equilateral hexagon in which all pairs of opposite sides are parallel. The triangle whose sides are extensions of segments \overline{AB} , \overline{CD} , and \overline{EF} has side lengths 200, 240, and 300. Find the side length of the hexagon.

Solution: (80).

6. (2024 AIME II Problem 6)

Alice chooses a set A of positive integers. Then Bob lists all finite nonempty sets B of positive integers with the property that the maximum element of B belongs to A . Bob's list has 2024 sets. Find the sum of the elements of A .

Solution: (55).

7. (2024 AIME II Problem 7)

Let N be the greatest four-digit positive integer with the property that whenever one of its digits is changed to 1, the resulting number is divisible by 7. Let Q and R be the quotient and the remainder, respectively, when N is divided by 1000. Find $Q + R$.

Solution: (699).

8. (2024 AIME II Problem 8)

Torus T is the surface produced by revolving a circle with radius 3 around an axis in the plane of the circle that is a distance 6 from the center of the circle.

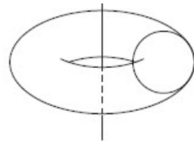


Figure 2: 2024 AIME II Problem 8

Let S be a sphere with radius 11. When T rests on the inside of S , it is internally tangent to S along a circle with radius r_i , and when T rests on the outside of S , it is externally tangent to S along a circle with radius r_o . The difference $r_i - r_o$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Solution: (127).

9. (2024 AIME II Problem 9)

There is a collection of 25 indistinguishable black chips and 25 indistinguishable white chips. Find the number of ways to place some of these chips in 2 unit cells of a 5×5 grid so that

- each cell contains at most one chip.
- all chips in the same row and all chips in the same column have the same color, and
- any additional chip placed on the grid would violate one or more of the previous two conditions.

Solution: (902).

10. (2024 AIME II Problem 10)

Let $\triangle ABC$ has incenter I , circumcenter O , inradius 6, and circumradius 13. Suppose that $\overline{IA} \perp \overline{OI}$. Find $AB \cdot AC$.

Solution: (468).

11. (2024 AIME II Problem 11)

Find the number of triples of nonnegative integers (a, b, c) satisfying $a + b + c = 300$ and

$$a^2b + a^2c + b^2a + b^2c + c^2a + c^2b = 6,000,000.$$

Solution: (601).

12. (2024 AIME II Problem 12)

Let $O(0, 0)$, $A\left(\frac{1}{2}, 0\right)$, and $B = \left(0, \frac{\sqrt{3}}{2}\right)$ be points in the coordinate plane. Let F be the family of segments \overline{PQ} of unit length lying in the first quadrant with P on the x -axis and Q on the y -axis. There is a unique point C on \overline{AB} , distinct from A and B , that does not belong to any segment from F other than \overline{AB} . Then $OC^2 = \frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

Solution:

(23).

13. (2024 AIME II Problem 13)

Let $\omega \neq 1$ be a 13th root of unity. Find the remainder when

$$\prod_{k=0}^{12} (2 - 2\omega^k + \omega^{2k})$$

is divided by 1000.

Solution: (321).

14. (2024 AIME II Problem 14)

Let $b \geq 2$ be an integer. Call a positive integer n b -eautiful if it has exactly two digits when expressed in base b , and these two digits sum to \sqrt{n} . For example, 81 is 13-eautiful because $81 = 63_{13}$ and $6 + 3 = \sqrt{81}$. Find the least integer $b \geq 2$ for which there are more than ten b -eautiful integers.

Solution: (211) .

15. (2024 AIME II Problem 15)

Find the number of rectangles inside a fixed regular dodecagon (12-gon) where each side of the rectangle lies on a side or on a diagonal of the dodecagon. The diagram below shows three of those rectangles.

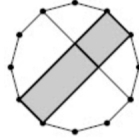


Figure 3: 2024 AIME II Problem 15

Solution: (315) .