

陈教授解读
2021 AIME II 真题，
核心解题技巧，
精细化答案
Professor Steven Chen



2021 AIME II 第一题

- Find the arithmetic mean of all the three-digit palindromes. (Recall that a palindrome is a number that reads the same forward and backward, such as 777 or 383).



2021 AIME II 第一题

- 核心解题技巧
 - 多重求和
 - 排列组合的乘法原理
- 都是陈教授教育学院培训课程AMC 12+中重点训练的内容



2021 AIME II 第一题

- 精细化答案

$$\frac{1100 \cdot 45}{90} = 550$$



2021 AIME II 第二题

- Equilateral triangle ABC has side length 840. Point D lies on the same side of line BC as A such that $\overline{BD} \perp \overline{BC}$. The line l through D parallel to line BC intersects sides \overline{AB} and \overline{AC} at points E and F , respectively. Point G lies on l , such that F is between E and G , $\triangle AFG$ is isosceles, and the ratio of the area of $\triangle AFG$ to the area $\triangle BED$ is 8:9. Find AF .



2021 AIME II 第二题

- 核心解题技巧
 - 方程方法解几何题
 - 含有正弦的面积公式
- 都是陈教授教育学院培训课程AMC 12+中重点训练的内容



2021 AIME II 第二题

- 精细化答案

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2021 AIME II 第三题

- Find the number of permutations x_1, x_2, x_3, x_4, x_5 of numbers 1, 2, 3, 4, 5 such that the sum of five products

$$x_1x_2x_3 + x_2x_3x_4 + x_3x_4x_5 + x_4x_5x_1 + x_5x_1x_2$$

- Is divisible by 3.



2021 AIME II 第三题

- 核心解题技巧
 - 5项里一定有3项包含3, 所以自然被3整除
 - 排列组合的乘法原理
- 都是陈教授教育学院培训课程AMC 12+中重点训练的内容



2021 AIME II 第三题

- 精细化答案

$$5 \cdot 8 \cdot 2 = 080$$



2021 AIME II 第四题

- There are real numbers a, b, c and d such that -20 is a root of $x^3 + ax + b$ and -21 is a root of $x^3 + cx^2 + d$. These two polynomials share a complex root $m + \sqrt{n} \cdot i$, where m and n are positive integers and $i = \sqrt{-1}$. Find $m + n$.



2021 AIME II 第四题

- 核心解题技巧
 - 多项式的共轭复数根
 - Vieta定理
- 都是陈教授教育学院培训课程AMC 12+中重点训练的内容



2021 AIME II 第四题

- 精细化答案

$$m = 10, n = 320 \rightarrow m + n = 330$$



2021 AIME II 第五题

- For positive real numbers s , let $\tau(s)$ denote the set of all obtuse triangles that have area s and two sides with lengths 4 and 10. The set of all s for which $\tau(s)$ is nonempty, but all triangles in $\tau(s)$ are congruent, is an interval $[a, b)$. Find $a^2 + b^2$.



2021 AIME II 第五题

- 核心解题技巧
 - 面积的正弦公式
 - 分类讨论
 - 判断钝角，直角，锐角的方法
 - 余弦定理
- 都是陈教授教育学院培训课程AMC 12+中重点训练的内容



2021 AIME I 第五题

- 精细化答案

$$[a, b) = [4\sqrt{21}, 20) \rightarrow a^2 + b^2 = 736$$



2021 AIME II 第六题

- For any finite set S , let $|S|$ denote the number of elements in S . Find the number of ordered pairs (A, B) , such that A and B are (not necessarily distinct) subsets of $\{1, 2, 3, 4, 5\}$ that satisfy

$$|A| \cdot |B| = |A \cap B| \cdot |A \cup B|$$



2021 AIME II 第六题

- 核心解题技巧
 - 集合的分割 (partition)
 - 排列组合中加法原理 (分类讨论) 和乘法原理 (分步做事) 的运用
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2021 AIME II 第六题

- 精细化答案

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2021 AIME II 第七题

- Let $a, b, c,$ and d be real numbers that satisfy the system of equations

$$\begin{aligned}a + b &= -3 \\ab + bc + ca &= -4 \\abc + bcd + cda + dab &= 14 \\abcd &= 30\end{aligned}$$

- There exists relatively prime positive integers m and n such that

$$a^2 + b^2 + c^2 + d^2 = \frac{m}{n}$$

- Find $m + n$.



2021 AIME II 第七题

- 核心解题技巧
 - 利用方程组的结构性解题
- 都是陈教授教育学院培训课程AMC 12+中重点训练的内容



2021 AIME II 第七题

- 精细化答案

$$(a, b) = (-5, 2) \text{ or } (2, -5), c = -2, d = \frac{3}{2}$$
$$\rightarrow a^2 + b^2 + c^2 + d^2 = \frac{141}{4}$$
$$\rightarrow m + n = 141 + 4 = 145$$



2021 AIME II 第八题

- An ant makes a sequence of moves on a cube, where a move consists of moving from one vertex to an adjacent vertex along an edge of a cube. Initially the ant is at a vertex of the bottom face of the cube and chooses one of the three adjacent vertices to move to as its first move. For all moves after the first move, the ant does not return to its previous vertex, but chooses to move to one of the other two vertices. All choices are selected at random so that each of the possible moves is equally likely. The probability that after exactly 8 moves the ant is at a vertex of the top face of the cube is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.



2021 AIME II 第八题

- 核心解题技巧
 - 递归方程求解概率问题
- 都是陈教授教育学院培训课程AMC 12+中重点训练的内容



2021 AIME II 第八题

- 精细化答案

$$\frac{17}{32} \rightarrow m + n = 17 + 32 = 049$$



2021 AIME II 第九题

- Find the number of ordered pairs (m, n) such that m and n are positive integers in the set $\{1, 2, \dots, 30\}$ and the greatest common divisor of $2^m + 1$ and $2^n - 1$ is not 1.



2021 AIME II 第九题

- 核心解题技巧
 - 最大公约数的性质
 - 补齐对称性求解排列组合题目
- 都是陈教授教育学院培训课程AMC 12+中重点训练的内容



2021 AIME II 第九题

- 精细化答案

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2021 AIME II 第十题

- Two spheres with radii 36 and one sphere with radius 13 are each externally tangent to the other two spheres and to two different planes P and Q . The intersection of planes P and Q is the line l . The distance from line l to the point where the sphere with radius 13 is tangent to plane P is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.



2021 AIME II 第十题

- 核心解题技巧
 - 球相切的处理方法
 - 立体几何问题转化为平面几何问题
 - 空间两点的距离
- 都是陈教授教育学院培训课程AMC 12+中重点训练的内容



2021 AIME II 第十题

- 精细化答案

$$\frac{312}{23} \rightarrow m + n = 312 + 23 = 335$$



2021 AIME II 第十一题

- A teacher was leading a class of four perfectly logical students. The teacher chose a set S of four integers and gave a different number in S to each student. Then the teacher announced in class that the numbers in S were four consecutive two-digit positive integers, that some number in S was divisible by 6, and a different number in S was divisible by 7. The teacher then asked if any of the students could deduce what S is, but in unison, all of the students replied no.
- However, upon hearing all students replied no, each student was able to determine the elements of S . Find the sum of all possible values of the greatest element of S .



2021 AIME II 第十一题

- 核心解题技巧
 - 逻辑分析
 - 同余方程组的求解
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2021 AIME II 第十一题

- 精细化答案

$$50 + 92 + 37 + 79 = 258$$



2021 AIME II 第十二题

- A convex quadrilateral has area 30 and side lengths 5, 6, 9, and 7, in that order. Denote by θ the measure of the acute angle formed by the diagonals of the quadrilateral. Then $\tan \theta$ can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.



2021 AIME II 第十二题

- 面积的正弦公式
- 余弦定理
- 利用对称性解具有结构性的方程组
- 都是陈教授教育学院培训课程AMC 12+中重点训练的内容



2021 AIME II 第十二题

- 精细化答案

$$\frac{40}{7} \rightarrow m + n = 40 + 7 = 047$$



2021 AIME II 第十三题

- Find the least positive integer n for which $2^n + 5^n - n$ is a multiple of 1000.



2021 AIME II 第十三题

- 核心解题技巧
 - 中国余数定理
 - 欧拉公式
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2021 AIME II 第十三题

- 精细化答案

$$n = 1000k + 797, k \in N \rightarrow 797$$



2021 AIME II 第十四题

- Let $\triangle ABC$ be an acute triangle with circumcenter O and centroid G . Let X be the intersection of the line tangent to the circumcircle of $\triangle ABC$ at A and the line perpendicular to \overline{GO} at G . Let Y be the intersection of lines XG and BC . Given that the measures of $\angle ABC$, $\angle BCA$, and $\angle XOY$ are in the ratio 13:2:17, the degree measure of $\angle BAC$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.



2021 AIME II 第十四题

- 核心解题技巧
 - 外心，重心的共同点：三角形边的中点
 - 四点共圆
 - 外心和对边张角的性质
- 都是陈教授教育学院培训课程AMC 12+中重点训练的内容



2021 AIME II 第十四题

- 精细化答案

$$\angle BAC = \frac{585^\circ}{7} \rightarrow m + n = 585 + 7 = 592$$



2021 AIME II 第十五题

- Let $f(n)$ and $g(n)$ be functions satisfying

$$f(n) = \begin{cases} \sqrt{n}, & \text{if } \sqrt{n} \text{ is an integer} \\ 1 + f(n + 1), & \text{otherwise} \end{cases}$$

$$g(n) = \begin{cases} \sqrt{n}, & \text{if } \sqrt{n} \text{ is an integer} \\ 2 + g(n + 2), & \text{otherwise} \end{cases}$$

- For positive integers n . Find the least possible integer n such that $\frac{f(n)}{g(n)} = \frac{4}{7}$.



2021 AIME II 第十五题

- 核心解题技巧
 - 列出 $f(n)$ 和 $g(n)$ 的通项表达式
 - 奇偶性分析
 - 整除
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2021 AIME II 第十五题

- 精细化答案

$$n = 2(18k^2 - 11k), k \in N, k \geq 3 \rightarrow 258$$

