2024 AIME I
Problems and Answer Key [*]

PROFESSOR CHEN EDUCATION PALACE

1. (2024 AIME I Problem 1)

Every morning Aya goes for a 9-kilometer-long walk and stops at a coffee shop afterwards. When she walks at a constant speed of s kilometers per hour, the walk takes her 4 hours, including t minutes spent in the coffer shop. When she walks at $s + 2$ kilometers per hour, the walk takes her 2 hours and 24 minutes, including t minutes spent in the coffee shop. Suppose Aya walks at $s + \frac{1}{2}$ kilometers per hour. Find the number of minutes the walk takes her, including the t minutes spent in the coffee shop.

Solution: $\boxed{(204)}$.

2. (2024 AIME I Problem 2)

There exist real numbers x and y, both greater than 1, such that $\log_x (y^x) = \log_y (x^{4y}) = 10$. Find xy.

Solution: $\boxed{(25)}$.

3. (2024 AIME I Problem 3)

Alice and Bob play the following game. A stack of n tokens lies before them. The players take turns with Alice going first. On each turn, the player removes either 1 token or 4 tokens from the stack. Whoever removes the last token wins. Find the number of positive integers n less than or equal to 2024 for which there exists a strategy for Bob that guarantees that Bob will win the game regardless of Alice's play.

Solution: $\boxed{(809)}$.

4. (2024 AIME I Problem 4)

Jen enters a lottery by selecting four distinct elements of $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Then four distinct elements of S are drawn at random. Jen wins a prize if at least two of her numbers are drawn, and she wins the grand prize if all four of her numbers are drawn. The probability that Jen wins the grand prize given that Jen wins a prize is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Solution: $\boxed{(116)}$.

5. (2024 AIME I Problem 5)

Rectangle $ABCD$ has dimensions $AB = 107$ and $BC = 16$, and rectangle $EFGH$ has dimensions $EF = 184$ and $FG = 17$. Points D, E, C, and F lie on line DF in that order, and A and H lie on opposite sides on line DF, as shown. Points A, D, H, and G lie on a common circle. Find CE.
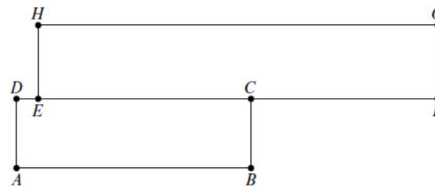

Figure 1: 2024 AIME I Problem 5

Solution: $\boxed{(104)}$.

6. (2024 AIME I Problem 6)

Consider the paths of length 16 that follow the lines from the lower left corner to the upper right corner on an 8×8 grid. Find the number of such paths that change direction exactly four times, as in the examples shown below.
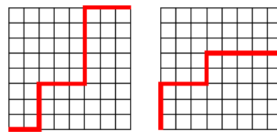

Figure 2: 2024 AIME I Problem 6

Solution: $\boxed{(294)}$.

7. (2024 AIME I Problem 7)

Find the greatest possible real part of

$$(75 + 117i)\, z + \frac{96 + 144i}{z},$$

where z is a complex number with $|z| = 4$. Here $i = \sqrt{-1}$.

Solution: (540) .

8. (2024 AIME I Problem 8)

Eight circles of radius 34 can be placed tangent to side $\overline{BC}$ of $\triangle ABC$ so that the circles are sequentially tangent to each other, with the first circle being tangent to $\overline{AB}$ and the last circle being tangent to $\overline{AC}$, as shown. Similarly, 2024 circles of radius 1 can be placed tangent to $\overline{BC}$ in the same manner. The inradius of $\triangle ABC$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.
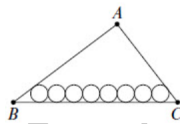

Figure 3: 2024 AIME I Problem 8

Solution: (197) .

9. (2024 AIME I Problem 9)

Let A, B, C, and D be points on the hyperbola $\frac{x^2}{20} - \frac{y^2}{24} = 1$ such that $ABCD$ is a rhombus whose diagonals intersect at the origin. Find the greatest real number that is less than BD^2 for all such rhombi.

Solution: (480) .

10. (2024 AIME I Problem 10)

Let $\triangle ABC$ have side lengths $AB = 5$, $BC = 9$, and $CA = 10$. The tangents to the circumcircle of $\triangle ABC$ at B and C intersect at point D, and $\overline{AD}$ intersects the circumcircle at $P \neq A$. The length of $\overline{AP}$ is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Solution: (113) .
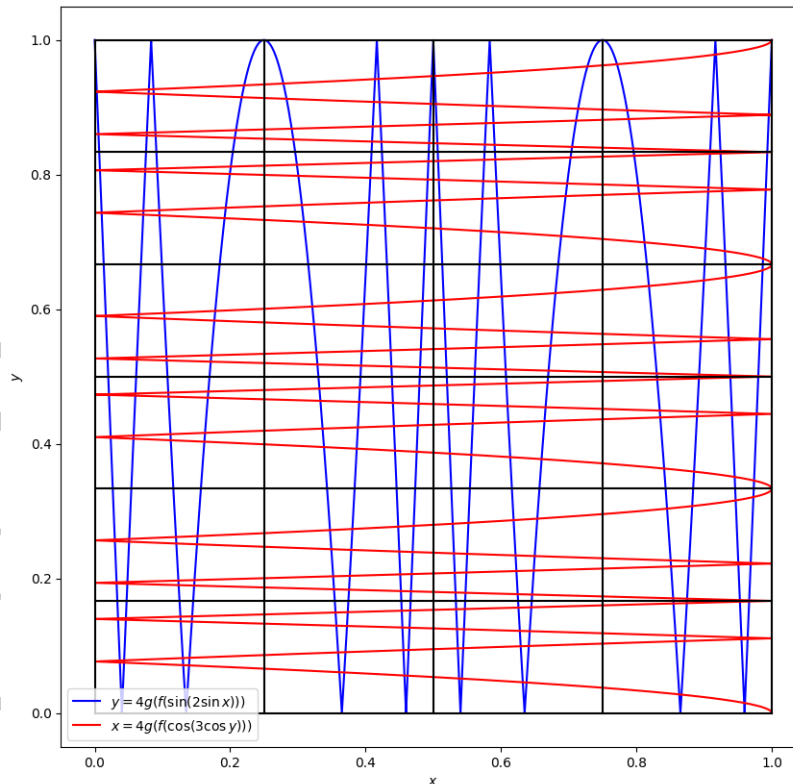
11. (2024 AIME I Problem 11)

Each vertex of a regular octagon is independently colored either red or blue with equal probability. The probability that the octagon can then be rotated so that all of the blue vertices move to positions where there had been red vertices is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Solution: (371) .

12. (2024 AIME I Problem 12)

Define $f(x) = \left| |x| - \frac{1}{2} \right|$ and $g(x) = \left| |x| - \frac{1}{4} \right|$. Find the number of intersections of the graphs of

$$y = 4g\left(f\left(\sin\left(2\pi x \right) \right) \right) \qquad \text{and } x = 4g\left(f\left(\cos\left(3\pi y \right) \right) \right).$$

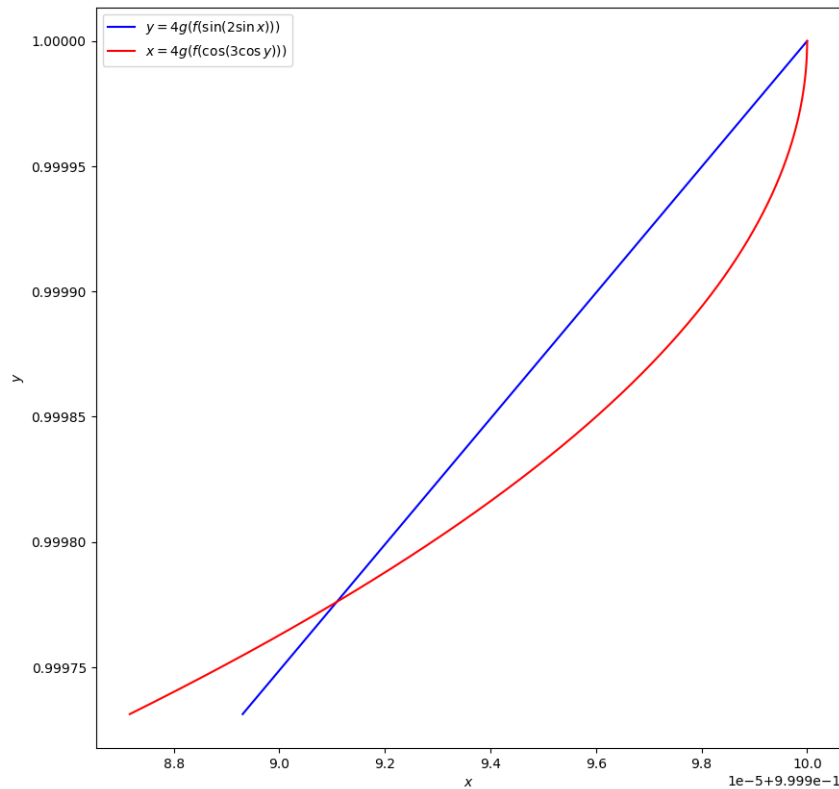**Figure 4:** 2024 AIME I Problem 12, Solution

Figure 5: 2024 AIME I Problem 12, two intersecting points near $(1,1)$

Solution: (385) .

13. (2024 AIME I Problem 13)

Let p be the least prime number for which there exists an integer n such that $n^4 + 1$ is divisible by p^2. Find the least positive integer m such that $m^4 + 1$ is divisible by p^2.

Solution: (110) .

14. (2024 AIME I Problem 14)

Let $ABCD$ be a tetrahedron such that $AB = CD = \sqrt{41}$, $AC = BD = \sqrt{80}$, and $BC = AD = \sqrt{89}$. There exists a point I inside the tetrahedron such that the distances from I to each of the faces of the tetrahedron are all equal. This distance can be written in the form $\frac{m\sqrt{n}}{p}$, where m, n, and p are positive integers, m and p are relatively prime, and n is not divisible by the square of any prime. Find $m + n + p$.

Solution: (104) .

15. (2024 AIME I Problem 15)

Let B be the set of rectangular boxes with surface area 54 and volume 23. Let r be the radius of the smallest sphere that can contain each of the rectangular boxes that are elements of B. The value of r^2 can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

Solution: (721) .