# 2022 AMC 12B
# Problems and Answer Key *

PROFESSOR CHEN EDUCATION PALACE

1. (2022 AMC 10B Problem 1)(2022 AMC 12B Problem 1)
Define $x \diamond y$ to be $|x-y|$ for all real numbers $x$ and $y$. What is the value of

$$(1 \diamond (2 \diamond 3)) - ((1 \diamond 2) \diamond 3)?$$

A. -2  B. -1  C. 0  D. 1  E. 2

**Solution:** **(A) -2**

2. (2022 AMC 10B Problem 2)(2022 AMC 12B Problem 2)
In rhombus $ABCD$, point $P$ lies on segment $\overline{AD}$ so that $\overline{BP} \perp \overline{AD}$, $AP = 3$, and $PD = 2$. What is the area of $ABCD$? (Note: The figure is not drawn to scale.)

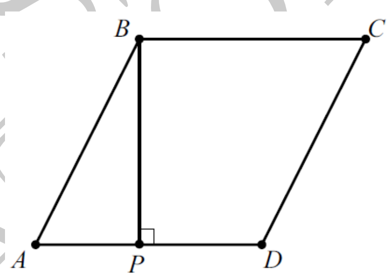


**Figure 1:** 2022 AMC 12B Problem 2

A. $3\sqrt{5}$  B. 10  C. $6\sqrt{5}$  D. 20  E. 25

**Solution:** **(D) 20**

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3. (2022 AMC 10B Problem 6)(2022 AMC 12B Problem 3)

   How many of the first ten numbers of the sequence $121, 11211, 1112111, \cdots$ are prime numbers?

   A. 0 B. 1 C. 2 D. 3 E. 4

   **Solution:** **(A) 0**

4. (2022 AMC 10B Problem 7)(2022 AMC 12B Problem 4)

   For how many values of the constant $k$ will the polynomial $x^2 + kx + 36$ have two distinct integer roots?

   A. 6 B. 8 C. 9 D. 14 E. 16

   **Solution:** **(B) 8**

5. (2022 AMC 12B Problem 5)

   The point $(-1, -2)$ is rotated $270°$ counterclockwise about the point $(3, 1)$. What are the coordinates of its new position?

   A. $(-3, 4)$ B. $(0, 5)$ C. $(2, -1)$ D. $(4, 3)$ E. $(6, -3)$

   **Solution:** **(B)** $(0, 5)$

6. (2022 AMC 10B Problem 8)(2022 AMC 12B Problem 6)

   Consider the following 100 sets of 10 elements each:

$$\begin{aligned}&\{1, 2, 3, \cdots, 10\},\\&\{11, 12, 13, \cdots, 20\},\\&\{21, 22, 23, \cdots, 30\},\\&\vdots\\&\{991, 992, 993, \cdots, 1000\}.\end{aligned}$$

   How many of these sets contain exactly two multiples of 7 ?

   A. 40 B. 42 C. 43 D. 49 E. 50

   **Solution:** **(B) 42**

7. (2022 AMC 10B Problem 10)(2022 AMC 12B Problem 7)

   Camila writes down five positive integers. The unique mode of these integers is 2 greater than their median, and the median is 2 greater than their arithmetic mean. What is the least possible value for the mode?

   A. 5 B. 7 C. 9 D. 11 E. 13

   **Solution:** **(D) 11**

8. (2022 AMC 12B Problem 8)

   What is the graph of $y^4 + 1 = x^4 + 2y^2$ in the coordinate plane?

   A. two intersecting parabolas B. two nonintersecting parabolas C. two intersecting circles D. a circle and a hyperbola E. a circle and two parabolas

   **Solution:** **(D) a circle and a hyperbola**

9. (2022 AMC 12B Problem 9)

   The sequence $a_0, a_1, a_2, \cdots$ is a strictly increasing arithmetic sequence of positive integers such that

   $$2^{a_7} = 2^{27} \cdot a_7.$$

   What is the minimum possible value of $a_2$?

   A. 8 B. 12 C. 16 D. 17 E. 22

   **Solution:** **(B) 12**

10. (2022 AMC 12B Problem 10)

    Regular hexagon $ABCDEF$ has side length 2. Let $G$ be the midpoint of $\overline{AB}$, and let $H$ be the midpoint of $\overline{DE}$. What is the perimeter of quadrilateral $GCHF$?

    A. $4\sqrt{3}$ B. 8 C. $4\sqrt{5}$ D. $4\sqrt{7}$ E. 12

    **Solution:** **(D)** $4\sqrt{7}$

11. (2022 AMC 12B Problem 11)

    Let $f(n) = \left(\frac{-1+i\sqrt{3}}{2}\right)^n + \left(\frac{-1-i\sqrt{3}}{2}\right)^n$, where $i = \sqrt{-1}$. What is $f(2022)$?

    A. -2 B. -1 C. 0 D. $\sqrt{3}$ E. 2

**Solution:** **(E) 2**

12. (2022 AMC 12B Problem 12)

Kayla rolls four fair 6-sided dice. What is the probability that at least one of the numbers Kayla rolls is greater than 4 and at least two of the numbers she rolls are greater than 2?

A. $\frac{2}{3}$ B. $\frac{19}{27}$ C. $\frac{59}{81}$ D. $\frac{61}{81}$ E. $\frac{7}{9}$

**Solution:** **(D)** $\frac{61}{81}$

13. (2022 AMC 10B Problem 16)(2022 AMC 12B Problem 13)

The diagram below shows a rectangle with side lengths 4 and 8 and a square with side length 5. Three vertices of the square lie on three different sides of the rectangle, as shown. What is the area of the region inside both the square and the rectangle?

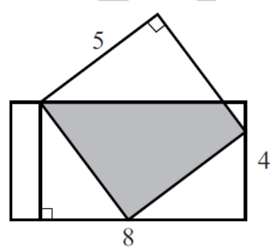


**Figure 2:** 2022 AMC 12B Problem 13

A. $15\frac{1}{8}$ B. $15\frac{3}{8}$ C. $15\frac{1}{2}$ D. $15\frac{5}{8}$ E. $15\frac{7}{8}$

**Solution:** **(D)** $15\frac{5}{8}$

14. (2022 AMC 12B Problem 14)

The graph of $y = x^2 + 2x - 15$ intersects the $x$-axis at points $A$ and $C$ and the $y$-axis at point $B$. What is $\tan(\angle ABC)$?

A. $\frac{1}{7}$ B. $\frac{1}{4}$ C. $\frac{3}{7}$ D. $\frac{1}{2}$ E. $\frac{4}{7}$

**Solution:** **(E)** $\frac{4}{7}$

15. (2022 AMC 10B Problem 17)(2022 AMC 12B Problem 15)

One of the following numbers is not divisible by any prime number less than 10. Which is it?

A. $2^{606}-1$  B. $2^{606}+1$  C. $2^{607}-1$  D. $2^{607}+1$  E. $2^{607}+3^{607}$

**Solution:** **(C)** $2^{607}-1$

16. (2022 AMC 12B Problem 16)

Suppose $x$ and $y$ are positive real numbers such that

$$x^y = 2^{64} \qquad \text{and} \qquad (\log_2 x)^{\log_2 y} = 2^7.$$

What is the greatest possible value of $\log_2 y$?

A. 3  B. 4  C. $3+\sqrt{2}$  D. $4+\sqrt{3}$  E. 7

**Solution:** **(C)** $3+\sqrt{2}$

17. (2022 AMC 12B Problem 17)

How many $4\times 4$ arrays whose entries are 0s and 1s are there such that the row sums (the sum of the entries in each row) are 1, 2, 3, and 4, in some order, and the column sums (the sum of the entries in each column) are also 1, 2, 3, and 4, in some order? For example, the array

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

satisfies the condition.

A. 140  B. 240  C. 336  D. 576  E. 624

**Solution:** **(D) 576**

18. (2022 AMC 10B Problem 19)(2022 AMC 12B Problem 18)

Each square in a $5\times 5$ grid is either filled or empty, and has up to eight adjacent neighboring squares, where neighboring squares share either a side or a corner. The grid is transformed by the following rules:

- Any filled square with two or three filled neighbors remains filled.
- Any empty square with exactly three filled neighbors becomes a filled square.

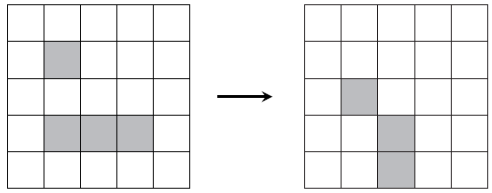


**Figure 3:** 2022 AMC 12B Problem 18 (1)

- All other squares remain empty or become empty.

A sample transformation is shown in the figure below.

Suppose the $5 \times 5$ grid has a border of empty squares surrounding a $3 \times 3$ subgrid. How many initial configurations will lead to a transformed grid consisting of a single filled square in the center after a single transformation? (Rotations and reflections of the same configuration are considered different.)

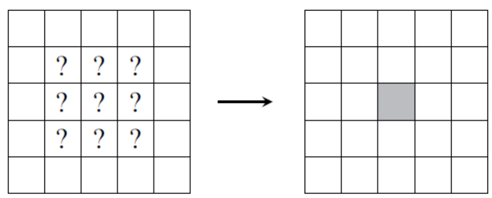


**Figure 4:** 2022 AMC 12B Problem 18 (2)

A. 14 B. 18 C. 22 D. 26 E. 30

**Solution:** **(C) 22**

19. (2022 AMC 12B Problem 19)

In $\triangle ABC$ medians $\overline{AD}$ and $\overline{BE}$ intersect at $G$ and $\triangle AGE$ is equilateral. Then $\cos(C)$ can be written as $\frac{m\sqrt{p}}{n}$, where $m$ and $n$ are relatively prime positive integers and $p$ is a positive integer not divisible by the square of any prime. What is $m+n+p$?

A. 44 B. 48 C. 52 D. 56 E. 60

**Solution:** **(A) 44**

20. (2022 AMC 10B Problem 21)(2022 AMC 12B Problem 20)

Let $P(x)$ be a polynomial with rational coefficients such that when $P(x)$ is divided by the polynomial $x^2+x+1$, the remainder is $x+2$, and when $P(x)$ is divided by the polynomial $x^2+1$, the remainder is $2x+1$. There is a unique polynomial of least degree with these two properties. What is the sum of the squares of the coefficients of that polynomial?

A. 10 B. 13 C. 19 D. 20 E. 23

**Solution:** **(E) 23**

21. (2022 AMC 10B Problem 22)(2022 AMC 12B Problem 21)

Let $S$ be the set of circles that are tangent to each of the three circles in the coordinate plane whose equations are $x^2+y^2=4$, $x^2+y^2=64$, and $(x-5)^2+y^2=3$. What is the sum of the areas of all the circles in $S$?

A. $48\pi$ B. $68\pi$ C. $96\pi$ D. $102\pi$ E. $136\pi$

**Solution:**
**(E)** $136\pi$

22. (2022 AMC 10B Problem 23)(2022 AMC 12B Problem 22)

Ant Amelia starts on the number line at 0 and crawls in the following manner. For $n=1,2,3,$ Amelia chooses a time duration $t_n$ and an increment $x_n$ independently and uniformly at random from the interval $(0,1)$. During the $n$th step of the process, Amelia moves $x_n$ units in the positive direction, using up $t_n$ minutes. If the total elapsed time has exceeded 1 minute during the $n$th step, she stops at the end of that step; otherwise, she continues with the next step, taking at most 3 steps in all. What is the probability that Amelia's position when she stops will be greater than 1?

A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. $\frac{2}{3}$ D. $\frac{3}{4}$ E. $\frac{5}{6}$

**Solution:** **(C)** $\frac{2}{3}$

23. (2022 AMC 10B Problem 25)(2022 AMC 12B Problem 23)

Let $x_0, x_1, x_2, \cdots$ be a sequence of numbers, where each $x_k$ is either 0 or 1. For each positive integer $n$, define

$$S_n = \sum_{k=0}^{n-1} x_k 2^k.$$

Suppose $7S_n \equiv 1 \pmod{2^n}$ for all $n \geq 1$. What is the value of the sum

$$x_{2019} + 2x_{2020} + 4x_{2021} + 8x_{2022}?$$

A. 6  B. 7  C. 12  D. 14  E. 15

**Solution:** **(A) 6**

24. (2022 AMC 12B Problem 24)

The figure below depicts a regular 7-gon inscribed in a unit circle.

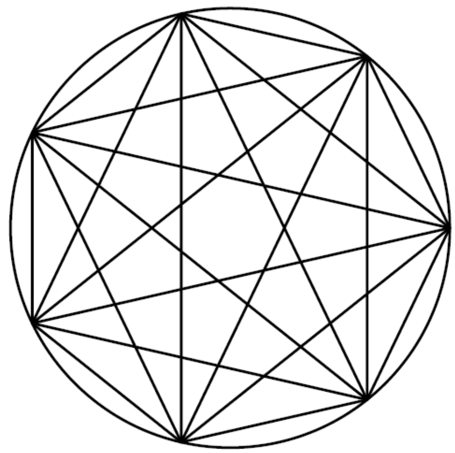

**Figure 5:** 2022 AMC 12B Problem 24

What is the sum of the 4th powers of the lengths of all 21 of its edges and diagonals?

A. 49  B. 98  C. 147  D. 168  E. 196

**Solution:** **(C) 147**

25. (2022 AMC 12B Problem 25)

Four regular hexagons surround a square with side length 1, each one sharing an edge with the square, as shown in the figure below. The area of the resulting 12-sided outer nonconvex polygon can be written as $m\sqrt{n}+p$, where $m$, $n$, and $p$ are integers and $n$ is not divisible by the square of any prime. What is $m+n+p$?

A. -12  B. -4  C. 4  D. 24  E. 32

**Solution:** **(B) -4**

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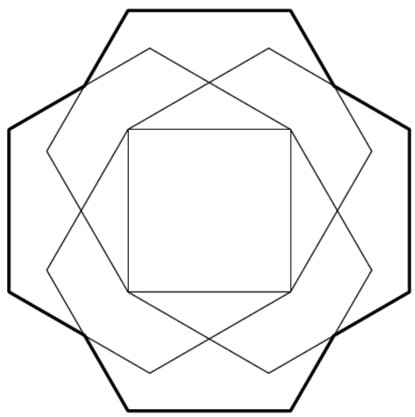

**Figure 6:** 2022 AMC 12B Problem 25