Advices on How to Prepare for The $F = ma$ Physics Contest

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1. Motivation of writing this note

We, PROFESSOR CHEN EDUCATION PALACE (www.professorchenedu.com) offer college counseling services and STEM Olympiad training courses for K-12 students.

One service that we offer is to help students prepare for the $F = ma$ physics contest. This contest is valuable in at least two-fold. First, students must qualify through $F = ma$ to establish their eligibility to take advanced physics Olympiad competitions, such as USAPhO and IPhO. Second, college admissions value a lot on students who perform well in $F = ma$. This is a strong and convincing evidence to show a student's critical thinking ability, problem solving skill and passion in physics and other subjects that require physics background (such as engineering, finance).

However, although many students realize the value of the $F = ma$ contest and have passions in taking this contest, they have many common misconceptions of how to effectively prepare for this contest. It is even more unfortunate that they do not even realize their misconceptions and thus put lots of effort in wrong directions. Let me name a few here.

1. **Common misconception 1: I only need to memorize some physics formulas. I can directly apply them during the contest. However, it is not necessary for me to understand how a formula is derived and under what conditions it holds.** Physics has many derivative results. However, different physics models (even for two models that only have slight difference) may require totally different underlying assumptions. Without understanding the assumptions used to derive a formula, you may use it in an incorrect setting where the assumptions are not satisfied. Furthermore, if you encounter a new problem and are unable to apply any of the formulas you know, you will not have an ability to develop a solution from scratch.

2. **Common misconception 2: I do not need to know advanced math (like calculus and advanced vector analysis) while preparing for $F = ma$.** It is possible to solve all problems in $F = ma$ without the use of calculus or advanced vector analysis. However, having these advanced math knowledge, such as the skill to do infinitesimal analysis, the capability of computing partial derivatives of a vector, and the fluency of doing inner product and cross product of vectors, can enable you to solve many problems correctly, easily and quickly.

3. **Common misconception 3: When I prepare for the $F = ma$ contest, to quickly get an answer and lighten my thinking burden, I should always exploit all multiple choices and get quick insights from them.** All problems in the $F = ma$ contest are

multiple choices. While taking the contest, it might be quicker to first take a look at given choices and derive intuition from them. However, this shortcut trick should not be heavily used when preparing for the contest. If you heavily rely on the given choices to solve a problem, you lose an opportunity to develop your ability of doing independent analysis of a problem. Therefore, if a problem maker develops a new physics model and designs choices that are hard for you to exploit, then you will be stuck.

Therefore, to avoid students to repeatedly make the same mistakes, I decide to write this note. In the rest of this note, I will share my advices on how to avoid the aforementioned mistakes and how to effectively prepare for the $F = ma$ contest.

2. Advice 1: Be equipped with calculus and vector analysis tools

Calculus and **vector analysis** are two important (or almost necessary) math tools to understand many physics concepts and solve many physics problems. The fundamental reason is that many physics quantities are defined with calculus and vectors. Therefore, the lack of mastering these math tools limits a student's understanding of physics concepts and ability to solve physics problems.

Let me use the following real $F = ma$ problems to justify my view.

2.1. Directly applying calculus formulas that define physics quantities

The following two problems can be directly solved by using integral formulas.

Example 1. *(2023 $F = ma$ Problem 1) A bead on a circular hoop with radius 2 m travels counter-clockwise for 10 s and completes 2.25 rotations, at which point it reaches the position shown.*

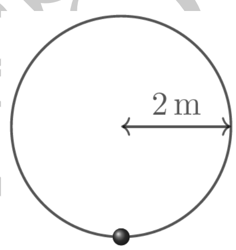

Figure 1: 2023 $F = ma$ Problem 1

In the past 10 s, what were its average speed and the direction of its average velocity?
A. $\frac{\sqrt{5}}{2}\ \frac{m}{s}$, ↙ *B.* $\frac{2\pi}{5}\ \frac{m}{s}$, ↙ *C.* $\frac{9\pi}{10}\ \frac{m}{s}$, ↙ *D.* $\frac{2\pi}{5}\ \frac{m}{s}$, ↘ *E.* $\frac{9\pi}{10}\ \frac{m}{s}$, ↘

If you are good at calculus and vector analysis, then this problem can be solved in one second. Denote by $\overrightarrow{v_t}$ the velocity at time t, then the average velocity over $[0, T]$ is simply

$$\frac{1}{T}\int_{t=0}^{T} \overrightarrow{v_t}\, dt = \overrightarrow{r_T} - \overrightarrow{r_0}.$$

Because $\overrightarrow{r_0}$ is on the left of the circle and $\overrightarrow{r_T}$ is at the bottom of the circle, the direction of the average velocity is ↘.

However, if you are not familiar with calculus and vector analysis, you have to do lots of computational work and you are also very likely to make a mistake. In addition, this approach works for all types of motions, not just the circular motion in this problem.

Let us look at one more real $F = ma$ problem.

Example 2. *(2023 $F = ma$ Problem 17) Three physical pendulums are built as shown. The first is a typical pendulum with a massless rope, and the second and third are made of uniform rods.*

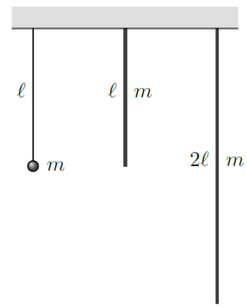

Figure 2: 2023 $F = ma$ Problem 17

What is the correct ranking of the moments of inertia I_1, I_2, and I_3 about the pivot points?
A. $I_1 > I_2 > I_3$ B. $I_3 > I_2 > I_1$ C. $I_1 = I_3 > I_2$ D. $I_2 > I_3 > I_1$ E. $I_3 > I_1 > I_2$

In this problem, we can directly take integral to compute the moments of inertia of rigid bodies 2 and 3.

2.2. Vector analysis and differential equations

Many physics quantities are vectors and continuously evolve over time. A precise (and also intuitive and transparent in many cases) way to characterize these processes and then solve problems is to establish differential equations for vectors. This requires students to have solid backgrounds in both calculus and vector analysis. Let me take the following real $F = ma$ contest problem as an example.

Example 3. *(2020 $F = ma$ Exam A Problem 24) A mass m is connected to one end of a zero-length spring with spring constant k. The other end of the spring is connected to a frictionless bearing mounted around a horizontal pole so that the mass can swing in a vertical circle of radius R around the pole. The setup is shown in the figure below. What is the vertical distance h between the center of the circular orbit and the axis of the pole? Assume that both the diameter of the pole and the rest length of the spring are negligible compared to R.*

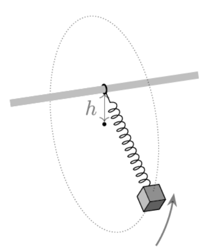

Figure 3: 2020 $F = ma$ Exam A Problem 24

A. $\sqrt{mgR/k}$ B. $R\sqrt{(R + mg/k)(R - mg/k)}$ C. $R - mg/k$ D. mg/k E. $\sqrt{R^2 - (mg/k)^2}$

I do not think this is an easy problem for a vast number of students. For the teaching purpose, I took a look at the official solution provided by AAPT[1]. I hoped to get a hint from the official solution to teach my students in an intuitive way. Unfortunately, the official solution lacks intuition and rigorous reasoning. For example, the solution states *"Consider the instant when the mass is moving vertically upward. In this instant the mass's acceleration is perfectly horizontal, ..."*. However, it fails to explain why the acceleration at this position has no vertical component. If this is so obvious, why does not the solution use a succinct and intuitive sentence to explain? If I cannot convince myself this is true after reading this sentence in the official solution, how do I expect my students to be convinced?

To intuitively and precisely understand the dynamics in this problem, we may directly establish differential equations with vectors without any fear of using these advanced math tools. My solution is as follows.

Solution. In the vertical plane with the circular motion, we denote by $\hat{\boldsymbol{i}}$ and $\hat{\boldsymbol{j}}$ the unit vectors that point towards the right and upward directions, respectively. We denote by $\hat{\boldsymbol{e}}_{\boldsymbol{r}}$ and $\hat{\boldsymbol{e}}_{\boldsymbol{\theta}}$ two unit vectors in the polar coordinate system of the same plane.

Thus,

$$\begin{aligned}\hat{\boldsymbol{i}} &= \cos\theta\ \hat{\boldsymbol{e}}_{\boldsymbol{r}} - \sin\theta\ \hat{\boldsymbol{e}}_{\boldsymbol{\theta}} \\ \hat{\boldsymbol{j}} &= \sin\theta\ \hat{\boldsymbol{e}}_{\boldsymbol{r}} + \cos\theta\ \hat{\boldsymbol{e}}_{\boldsymbol{\theta}}\end{aligned}$$

At a position with polar coordinates (R, θ), the net force is

$$\begin{aligned}\overrightarrow{\boldsymbol{F}} &= -mg\ \hat{\boldsymbol{j}} + k\left(h\ \hat{\boldsymbol{j}} - R\ \hat{\boldsymbol{e}}_{\boldsymbol{r}}\right) \\ &= \left((kh - mg)\sin\theta - R\right)\ \hat{\boldsymbol{e}}_{\boldsymbol{r}} + (kh - mg)\cos\theta\ \hat{\boldsymbol{e}}_{\boldsymbol{\theta}}.\end{aligned}$$

The acceleration is

$$\overrightarrow{\boldsymbol{a}} = -\dot{\theta}^2 R\ \hat{\boldsymbol{e}}_{\boldsymbol{r}} + \ddot{\theta} R\ \hat{\boldsymbol{e}}_{\boldsymbol{\theta}}.$$

Following from Newton's second law that $\overrightarrow{\boldsymbol{F}} = m\overrightarrow{\boldsymbol{a}}$, we have

$$\left((kh - mg)\sin\theta - R\right)\ \hat{\boldsymbol{e}}_{\boldsymbol{r}} + (kh - mg)\cos\theta\ \hat{\boldsymbol{e}}_{\boldsymbol{\theta}} = -\dot{\theta}^2 R\ \hat{\boldsymbol{e}}_{\boldsymbol{r}} + \ddot{\theta} R\ \hat{\boldsymbol{e}}_{\boldsymbol{\theta}}.$$

Thus,

$$(kh - mg)\sin\theta - R = -\dot{\theta}^2 R \qquad (1)$$

$$(kh - mg)\cos\theta = \ddot{\theta} R \qquad (2)$$

For (1), taking derivative with respect to time t on both sides, we get

$$(kh - mg)\cos\theta\ \dot{\theta} = -2\dot{\theta}\ddot{\theta} R.$$

Dividing both sides by $\dot{\theta}$, we get

$$(kh - mg - R)\cos\theta = -2\ddot{\theta} R. \qquad (1')$$

Taking $(2)\cdot 2 + (1')$, we get

$$3\,(kh - mg)\cos\theta = 0.$$

Therefore, $h = \frac{mg}{k}$. ■

[1] https://www.aapt.org/Common/upload/2020-Fma-Exam-A_solutions.pdf

As you can see, by using calculus and vector tools, each step in my solution is transparent, intuitive, rigorous in physics and math. Although it sounds like I used much more complicated math tools than the AAPT official solution, this makes the thinking and analysis process smooth and rigorous.

2.3. Infinitesimal analysis

Many physics problems are with continuous time (classical mechanics) and continuous space (waves in quantum mechanics and electromagnetic fields). Infinitesimal analysis is a necessary skill to establish equations for these problems. Therefore, $F = ma$ contest frequently tests students the skills of doing infinitesimal analysis.

At a high level, we can understand infinitesimal analysis in $F = ma$ as an application of using the essence of mathematical analysis (such as differentiation, limit, Taylor's expansion) to build up and solve physics models. For example, we need to know how to do Taylor's expansions of trig functions $\cos x$ and $\sin x$ with small x and binomial functions $(1+x)^k$ with small x. All ideas and results are from calculus. Therefore, the calculus background is quite valuable, and even indispensable. Below is an example of the infinitesimal analysis.

Example 4. *(2020 $F = ma$ Exam A Problem 25) A ball of negligible radius and mass m is connected to two ideal springs. Each spring has rest length l_0. The springs are connected to the ball inside a box of height $2l_0$, and the ball is allowed to come to equilibrium, as shown. Under what condition is this equilibrium point stable with respect to small horizontal displacements?*

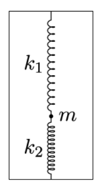

Figure 4: 2020 $F = ma$ Exam A Problem 25

A. $k_1 > k_2$
B. $k_2 > k_1$
C. $k_1 - k_2 > mg/l_0$
D. $k_1 k_2/(k_1 + k_2) > mg/l_0$
E. $k_1 k_2/(k_1 - k_2) > mg/l_0$

I first present my solution. My comments then follow up.

Solution. Denote by $y_0 + l_0$ the distance between the ball that is in equilibrium and the ceiling. In equilibrium, we have

$$mg = k_1 y_0 + k_2 y_0.$$

Hence, $y_0 = \frac{mg}{k_1+k_2}$.

Denote by x the ball's horizontal displacement from the equilibrium. Thus, the total energy is

$$E(x) = mg(y_0 - l_0) + \frac{1}{2}k_1\left(\sqrt{x^2 + (l_0+y_0)^2} - l_0\right)^2 + \frac{1}{2}k_2\left(\sqrt{x^2 + (l_0-y_0)^2} - l_0\right)^2.$$

To find the condition that the equilibrium is stable, it is equivalent to find the condition that $\frac{d^E(x)}{dx^2}|_{x=0} > 0$. However, it is a big mess to directly take twice derivatives of $E(x)$.

Since we are doing infinitesimal analysis for small $|x|$ up to the second order, we can do Taylor's expansion of $E(x)$ and find the second order coefficient.

We use the following result by doing Taylor's expansion of $E(x)$:

$$\sqrt{1+u} = 1 + \frac{1}{2}u + o\left(u^2\right).$$

Thus, the second order term of the Taylor's expansion of $E(x)$ is

$$\frac{1}{2}(k_1+k_2) - \frac{l_0}{2}\left(\frac{k_1}{l_0+y_0} + \frac{k_2}{l_0-y_0}\right).$$

This can be further simplified as

$$\begin{aligned}\frac{1}{2}(k_1+k_2) - \frac{l_0}{2}\left(\frac{k_1}{l_0+y_0} + \frac{k_2}{l_0-y_0}\right) &= \frac{y_0}{2}\left(\frac{k_1}{l_0+y_0} + \frac{k_2}{l_0-y_0}\right) \\ &= \frac{y_0}{2(l_0+y_0)(l_0-y_0)}\left((k_1-k_2)l_0 - (k_1+k_2)y_0\right) \\ &= \frac{y_0}{2(l_0+y_0)(l_0-y_0)}\left((k_1-k_2)l_0 - mg\right). \qquad (1)\end{aligned}$$

The third equality follows from our previously derived result that $y_0 = \frac{mg}{k_1+k_2}$.

Therefore, the equilibrium is stable (equivalently, term (1) is positive if and only if $k_1 - k_2 > \frac{mg}{l_0}$.

Therefore, the answer is **(C)**. ■

In my solution, because of the non-linearity of the energy function $E(x)$, directly computing its second order derivative requires lots of work. However, it is much easier of doing the infinitesimal analysis for small $|x|$ (Taylor's expansion) to find the coefficient of x^2. In this analysis, I used the property that $\sqrt{1+u} = 1 + \frac{1}{2}u + o\left(u^2\right)$. Therefore, this example shows us the power of infinitesimal analysis. Again, if you want to proficiently use this method, you need to have a solid background in calculus.

2.4. Vector analysis: Beyond orthogonal decomposition

For vector analysis, some students have limited knowledge of only knowing how to decompose a vector into component vectors and then do scalar analysis on each axis. However, this is far from being sufficient. Advanced vector analysis skills are also needed.

Example 5. *Derive a formula of the angular moment of a point mass with mass m, position $\overrightarrow{\boldsymbol{r}}$, and angular velocity $\overrightarrow{\boldsymbol{\omega}}$.*

Solution. We have

$$\begin{aligned}\overrightarrow{\boldsymbol{L}} &= \overrightarrow{\boldsymbol{r}} \times m\overrightarrow{\boldsymbol{v}} \\ &= m\overrightarrow{\boldsymbol{r}} \times (\overrightarrow{\boldsymbol{\omega}} \times \overrightarrow{\boldsymbol{r}}) \\ &= m\left((\overrightarrow{\boldsymbol{r}} \cdot \overrightarrow{\boldsymbol{r}})\overrightarrow{\boldsymbol{\omega}} - (\overrightarrow{\boldsymbol{r}} \cdot \overrightarrow{\boldsymbol{\omega}})\overrightarrow{\boldsymbol{r}}\right) \\ &= m\left(r^2\overrightarrow{\boldsymbol{\omega}} - (\overrightarrow{\boldsymbol{r}} \cdot \overrightarrow{\boldsymbol{\omega}})\overrightarrow{\boldsymbol{r}}\right).\end{aligned}$$

The third equality follows from the vector triple product that

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = (\vec{a} \cdot \vec{c})\,\vec{b} - \left(\vec{a} \cdot \vec{b}\right)\vec{c}.$$

■

As we can see from this result, the angular momentum is not always in the same direction of the angular velocity, and the magnitude of the angular momentum is not always in the form of $mr^2\omega$. We may have the second term $m\left(\vec{r} \cdot \vec{\omega}\right)\vec{r}$.

When you read up to here, you may be confused why we have this second term and why you may have never seen it in your physics study. To completely understand why this second term disappears in many high school physics textbooks, we need to find a sufficient condition for the general formula above, such that this second term disappears under this condition. A sufficient condition is $\vec{r} \cdot \vec{\omega} = 0$. This means $\vec{r} \perp \vec{\omega}$. Therefore, if we see a problem that a point mass is doing a rotational motion and the reference point for computing torques and angular momentum is in the same plane of the rotational motion (this is mostly seen in high school physics study), then we have $\vec{r} \perp \vec{\omega}$. That being said, the second term in the general formula above disappears.

Now, let us imagine that in an upcoming $F = ma$ contest, you are asked to solve a problem where the above relative position no longer holds, such as using the pivot in 2023 $F = ma$ Problem 6 as a reference point and analyzing torques and angular momentum. Then you cannot use the special case formula that drops off the second term above. You have to use the complete formula. The way to fully understand this formula and correctly use it is to have a solid knowledge of doing the above vector analysis with cross product and inner product.

Next, we look at a real $F = ma$ problem and see how the vector analysis helps us solve the problem.

Example 6. *(2023 $F = ma$ Problem 23) An astronomer on Earth, which is a distance $L_\odot$ from the Sun, observes a star and galaxy. The star is a distance $L_S \gg L_\odot$ away, and the galaxy is much further away than the star. Throughout the year, the angular distance between the star and galaxy appears to vary, reaching a minimum of 2 arcseconds on January 1st and a maximum of 2.5 arcseconds on July 1st. (One degree is equal to 3600 arcseconds.) Assume the Sun, star, and galaxy do not move relative to each other, and that the Earth's orbit lies within their plane. What is the ratio $L_s/L_\odot$?*

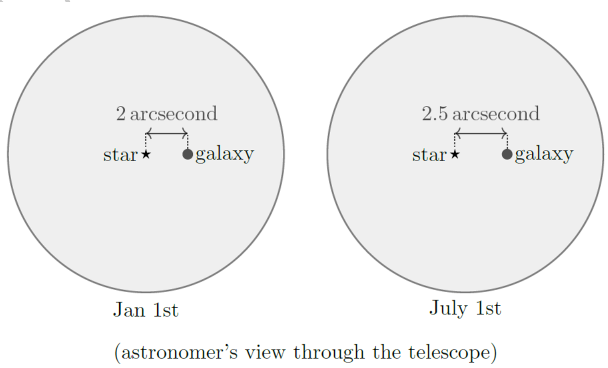

Figure 5: 2023 $F = ma$ Problem 23

A. 1.4×10^4 *B.* 7×10^4 *C.* 4×10^5 *D.* 8×10^5 *E.* 4×10^6

Solution. Denote by O, E, S, G the positions of the sun, the earth, the star, and the galaxy, respectively. Denote $\theta = \angle SOG$. Denote $\phi = \angle GOE$.

In the rest of analysis, we denote by $\hat{\boldsymbol{r}} = \frac{\overrightarrow{\boldsymbol{r}}}{|\overrightarrow{\boldsymbol{r}}|}$ the unit vector in the direction of $\overrightarrow{\boldsymbol{r}}$.

We have

$$\begin{aligned}
\cos \angle SEG &= \frac{\overrightarrow{ES} \cdot \overrightarrow{EG}}{ES \cdot EG} \\
&= \frac{\left(\overrightarrow{OS} - \overrightarrow{OE}\right) \cdot \left(\overrightarrow{OG} - \overrightarrow{OE}\right)}{|\overrightarrow{OS} - \overrightarrow{OE}| \cdot |\overrightarrow{OG} - \overrightarrow{OE}|} \\
&= \frac{\left(\widehat{OS} - \frac{OE}{OS}\widehat{OE}\right) \cdot \left(\widehat{OG} - \frac{OE}{OG}\widehat{OE}\right)}{\left|\widehat{OS} - \frac{OE}{OS}\widehat{OE}\right| \cdot \left|\widehat{OG} - \frac{OE}{OG}\widehat{OE}\right|} \\
&\approx \frac{\left(\widehat{OS} - \frac{OE}{OS}\widehat{OE}\right) \cdot \widehat{OG}}{\left|\widehat{OS} - \frac{OE}{OS}\widehat{OE}\right|} \\
&= \frac{\cos\theta - \frac{OE}{OS}\cos\phi}{\sqrt{1 - 2\frac{OE}{OS}\cos(\theta+\phi) + \left(\frac{OE}{OS}\right)^2}} \\
&\approx \left(\cos\theta - \frac{OE}{OS}\cos\phi\right)\left(1 + \frac{OE}{OS}\cos(\theta+\phi)\right) \\
&\approx \cos\theta - \frac{OE}{OS}\cos\phi + \frac{OE}{OS}\cos(\theta+\phi) \\
&= \cos\theta - 2\frac{OE}{OS}\sin\frac{\theta}{2}\sin\left(\phi + \sin\frac{\theta}{2}\right).
\end{aligned}$$

All approximations above follow from the condition that $OG \gg OS \gg OE$.

We notice that $\angle SEG$ is small all year around. Thus, θ is also small. Taking approximations that $\cos x \approx 1 - \frac{x^2}{2}$ and $\sin x \approx x$ for small x, the equation above can be approximated as

$$1 - \frac{(\angle SEG)^2}{2} \approx 1 - \frac{\theta^2}{2} - \frac{OE}{OS}\theta \sin\left(\phi + \sin\frac{\theta}{2}\right).$$

Denote $\alpha = \max \angle SEG = 2.5$ *arcseconds* and $\beta = \min \angle SEG = 2$ *arcseconds*. We notice that $\sin\left(\phi + \sin\frac{\theta}{2}\right) \in [-1, 1]$. Thus, we have

$$\frac{\theta^2}{2} + \frac{OE}{OS}\theta = \frac{\alpha^2}{2} \qquad (1)$$
$$\frac{\theta^2}{2} - \frac{OE}{OS}\theta = \frac{\beta^2}{2} \qquad (2)$$

Taking $\sqrt{(1)+(2)}$, we get

$$\theta = \sqrt{\frac{\alpha^2 + \beta^2}{2}} \qquad (3).$$

Taking $\frac{(1)-(2)}{2(3)}$, we get

$$\frac{OE}{OS} = \frac{\alpha^2 - \beta^2}{2\sqrt{2\left(\alpha^2 + \beta^2\right)}}.$$

Because $OS \gg OE$,

$$
\begin{aligned}
\frac{L_s}{L_\odot} &\approx \frac{OS}{OE} \\
&= \frac{2\sqrt{2\left(\alpha^2+\beta^2\right)}}{\alpha^2-\beta^2} \\
&= \frac{2\sqrt{2\left(2.5^2+2^2\right)}}{2.5^2-2^2} \cdot \frac{3600 \cdot 180}{\pi} \\
&= 8.3 \times 10^5.
\end{aligned}
$$

■

As you can see from the above solution, it is very difficult to purely use our intuitions to find correct geometric positions of the sun, the star, the galaxy and the earth, at which the angle between the star and the galaxy from the earth's view is maximized or minimized. Therefore, the safest way is to do brute force by using the formula of the inner product of two vectors to algebraically compute the cosine value of the above angle.

In addition to the comment above about the usefulness of vector analysis tools, I also want to comment on the infinitesimal analysis of this problem. In my solution, I did the first-order approximations of $\sin x$ and $(1+x)^k$ and the second-order approximation of $\cos x$. All of these come from my knowledge in calculus about the Taylor's expansions.

2.5. Measurement with uncertainties

In the $F = ma$ contest, there are lots of problems about experimental measurements with uncertainties, such as the following two real $F = ma$ problems.

Example 7. *(2020 $F = ma$ Exam A Problem 23) Steve determines the spring constant k of a spring by applying a force F to it and measuring the change in length Δx. The tools he uses to measure F and Δx both have a constant absolute uncertainty, leading to an uncertainty in k of δk_S. If Tiffany measures the same spring constant with the same tools but by using a force that is five times larger, what will her uncertainty in k be in terms of δk_S?*

A. $\delta k_T = 0.04\delta k_S$
B. $\delta k_T = 0.08\delta k_S$
C. $\delta k_T = 0.2\delta k_S$
D. $\delta k_T = 0.4\delta k_S$
E. $\delta k_T = 0.5\delta k_S$

Example 8. *(2023 $F = ma$ Problem 19) A student sets up a simple pendulum, measures its length to be (0.50 ± 0.01) m, and observes a period of oscillation of (1.4 ± 0.1) s. Using this data, the student computes $g = 10.1$ m/s^2. What is the uncertainty of this measurement?*

A. 0.7 m/s^2 *B.* 1.2 m/s^2 *C.* 1.4 m/s^2 *D.* 1.9 m/s^2 *E.* 2.7 m/s^2

These problems ask us to compute the uncertainty of a dependent variable that is a function of some independent variables that are all directly measured with uncertainties. The general form of this type of problems is as follows. Let $Y = f(X_1, \cdots, X_n)$. For each $i \in \{1, \cdots, n\}$, if the measurement of X_i is $x_i \pm \sigma_i$ and the measurement uncertainties of $X_1, \cdots, X_n$ are independent, then what is the measurement uncertainty of Y, denoted as σ_Y.

The general formula to compute σ_Y is the following one:

$$\sigma_Y = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial Y}{\partial X_i} \bigg|_{(X_1, \cdots, X_n) = (x_1, \cdots, x_n)} \right)^2 \sigma_i^2}.$$

Although this formula can be used to solve all illustrated problems above, using it requires students to know how to do partial derivatives. Otherwise, students have to memorize lots of special forms of this formula. However, these cannot be exhaustive. In addition, students can only mechanically memorize those special form formulas without knowing where they come from. As a result, if the upcoming contest makes a new function $f(\cdot)$ that students have never seen, they do not have a special form formula to use and thus they will be stuck.

3. Advice 2: Figure out how to prove all formulas that you have seen

Many students have a common bad habit while studying physics. While seeing a new physics problem, their first reaction is not to understand the physics process and start from the basic physics principles (such as Newton's laws, conservation of momentum) to establish equations. By contrast, they just lazily apply some derivative results that they have memorized to solve a new problem.

However, not all problems can be solved by directly quoting the derivative results learned somewhere. There are always new problems that the derivative results you know do not fit.

Unfortunately, a vast number of students have no idea how to start from the basic physics principles to establish equations and then finally solve a problem. By talking to quite a few such students, I realize why they cannot do this. When they took their high school physics courses, if they saw a new result and the teacher or the textbook did not prove the result for them, they did not try to understand how the result was derived and under what conditions the result holds or does not hold. Without such training, we cannot expect them to suddenly develop ability in a physics contest.

Let me use the following real $F = ma$ problems as examples to support my view.

Example 9. *(2023 $F = ma$ Problem 22) Two springs with different spring constants are connected in three ways, as shown.*

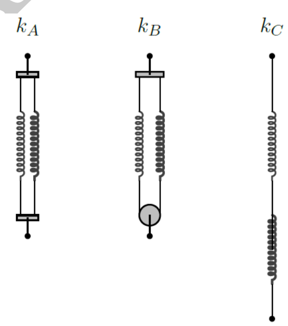

Figure 6: 2023 $F = ma$ Problem 22

In the second case, the springs are connected to opposite ends of a string, which runs under a massless frictionless pulley. In each case, the two springs act like a single spring with an effective spring constant k_A, k_B, or k_C. Which of the following is correct?

A. $k_A > k_B > k_C$
B. $k_A > k_C > k_B$
C. $k_C > k_B > k_A$
D. $k_C > k_A > k_B$
E. $k_B > k_A > k_C$

Before solving this problem in the contest, some student might have learned some formulas of the effective spring constants of some types of spring configurations, such as the parallel (the left on in the figure) or serial (the right one in the figure) configurations. However, those formulas are not exhaustive. Not so many students had seen the formula of the effective spring constant for the configuration in the middle of the figure. For such students, when they saw this problem, they had no idea how to derive the formula from the basic physics principles for the middle configuration.

This is primarily due to their learning habits. When they saw the formulas for the parallel and serial configurations, they only memorized those formulas. However, they did not ask themselves how those formulas were derived. They did not try to figure out how to prove those formulas. Due to the lack of such rigorous training of deriving derivative formulas from scratch, they felt challenge of deriving a formula for a new problem (the middle configuration) they did not see before.

To fundamentally address this issue, when students learn the derivative formulas of the effective spring constants for some special spring configurations, they need to stop for a second and ask themselves how those formulas are derived. For example, for any type of spring configurations, to compute the effective spring constant, a student should always make a displacement of an object. After that, the student needs to analyze the effect of this displacement on the length change of every spring. The student then compute the joint restoring forces applied by all springs on the object. The student can then derive the effective spring constant by taking the ratio of the joint restoring force to the displacement.

Next, let us look at the second real contest problem in $F = ma$.

Example 10. *(2020 $F = ma$ Exam A Problem 22) A collision occurs between two masses. In each inertial reference frame, one can compute the change in total momentum $\Delta \boldsymbol{P}$ and the change in total kinetic energy ΔK due to the collision. Which of the following is true?*

A. $\Delta \boldsymbol{P}$ *and* ΔK *do not depend on the frame.*

B. $\Delta \boldsymbol{P}$ *and* ΔK *do not depend on the frame for perfectly elastic collisions, but* $\Delta \boldsymbol{P}$ *may depend on the frame for inelastic collisions.*

C. $\Delta \boldsymbol{P}$ *and* ΔK *do not depend on the frame for perfectly elastic collisions, but* ΔK *may depend on the frame for inelastic collisions.*

D. $\Delta \boldsymbol{P}$ *and* ΔK *do not depend on the frame for perfectly elastic collisions, but both may depend on the frame for inelastic collisions.*

E. $\Delta \boldsymbol{P}$ *and* ΔK *may both depend on the frame, for both perfectly elastic and inelastic collisions.*

I went over AAPT's official solution[2]. However, this official solution is too descriptive. In addition, if an upcoming contest asks students another collision problem that requires quantitative analysis, students do not know what to do from reading this solution.

To solve this problem in a crystal clear way, for the physics quantities of both momentum and kinetic energy, a student needs to have a thorough understanding of the relationship of each quantity in an arbitrary inertial reference frame and the system's center of mass reference frame. That is, for a system in an inertial reference,

[2] `https://www.aapt.org/Common/upload/2020-Fma-Exam-A_solutions.pdf`

1. (Momentum) The system's total momentum is always equal to the mass center's momentum. They evolve at the same rate. In the center of mass reference frame, the total momentum is always zero.

2. (Kinetic energy) The system's total kinetic energy is equal to the mass center's kinetic energy plus the total kinetic energy in the center of mass reference frame.

The best way to memorize the theorems above and know the conditions that they hold is to self-prove them from scratch. By going over the whole proof, students can understand why these results hold only for the center of mass, but not any other point in the system. Students can also understand why these results always hold even if the center of mass reference frame is not an inertial reference frame (when the system's external net force is not zero).

To further convince you the value of proving all derivative results like those listed above, let me give you another real contest example.

Example 11. *(2023 $F = ma$ Problem 21) A smooth ring of radius R and mass m lies on a frictionless surface. A point mass, also of mass m, is placed just inside the ring and given a speed v tangent to the inner surface of the ring. How long does it take for the point mass to return to its initial position relative to the ring?*

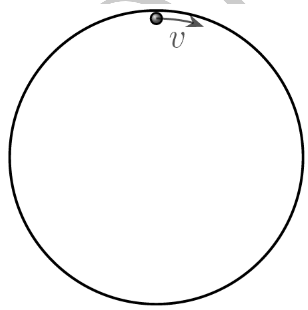

Figure 7: 2023 $F = ma$ Problem 21

A. $\frac{\pi R}{v}$ *B.* $\frac{\sqrt{2}\pi R}{v}$ *C.* $\frac{2\pi R}{v}$ *D.* $\frac{2\sqrt{2}\pi R}{v}$ *E.* $\frac{4\pi R}{v}$

This is a more quantitative version of Example 10. However, the AAPT's official solution[3] lacks rigor and sufficient reasoning. There is a big gap between two successive sentences in this solution. It is not easy for students to justify each sentence and fill out the gap between two successive sentences.

Therefore, if students want to independently solve this problem, they need to know how to start from the basic physics principles and make the analysis in each step to be rigorous and justifiable.

Let me present my solution below. Prior to it, I need to first introduce and prove a lemma used in my solution.

Lemma 1. *A point mass and a rigid body move on the same plane. There is a frictionless collision between them. If the normal direction of the surface of the rigid body at the collision point passes through the center of mass of the rigid body, then in the rigid body's center of mass reference frame, the rigid body's angular momentum about its center of mass remains the same after collision.*

[3] `https://aapt.org/physicsteam/2023/upload/2023_F-ma_Solutions_v3.pdf`

Proof. Denote by C the center of mass of the rigid body. Denote by A the collision point. Denote by $\hat{\boldsymbol{e}}$ the unit vector in the normal direction of the surface of the rigid body at point A. Denote by $\overrightarrow{\boldsymbol{I}}$ the impulse the point mass applies to the rigid body during the collision.

Because the collision happens on direction $\hat{\boldsymbol{e}}$ and the collision is frictionless, $\overrightarrow{\boldsymbol{I}}$ and $\hat{\boldsymbol{e}}$ are on the same line. Because $\hat{\boldsymbol{e}}$ is on the same line of $\overrightarrow{CA}$, we must have that $\overrightarrow{\boldsymbol{I}}$ and $\overrightarrow{CA}$ are on the same line.

Because the rigid body is doing a planar motion, in the rigid body's center of mass reference frame, the change of the angular momentum about the center of mass is

$$\begin{aligned}\Delta \overrightarrow{\boldsymbol{L}} &= \overrightarrow{CA} \times \overrightarrow{\boldsymbol{I}} \\ &= 0,\end{aligned}$$

where the second equality follows from the property that $\overrightarrow{\boldsymbol{I}}$ and $\overrightarrow{CA}$ are on the same line. ■

Now, I am ready to present my solution.

Solution. Denote by A the point mass and O the center of the ring. Because mass is uniformly distributed on the ring, O is the center of mass of the ring.

Denote by C the center of mass of the system that consists of the point mass and the ring (please note that this is different from O that is the center of mass of the ring). Now, we do our analysis in the system's center of mass reference frame.

In the initial position, C is the midpoint of A and O, with $AC = OC = \frac{R}{2}$.

Because the point mass and the ring have the same mass, in the system's center of mass reference frame, the point mass has the initial velocity $\frac{\overrightarrow{v}}{2}$ and the center of the ring O has the initial velocity $-\frac{\overrightarrow{v}}{2}$. The ring has no initial rotation about O.

We notice that the normal direction at any point on the ring passes through O and any collision between the point mass and the ring is frictionless. Thus, the lemma above implies that the ring never rotates about O irrespectively of how the point mass and the ring collide over time. That is, we only need to consider the ring's translational move.

At time t, for the point mass, we denote by $\overrightarrow{\boldsymbol{r_t}}$ its position about C and $\overrightarrow{\boldsymbol{v_t}}$ its velocity. For the center of the ring O, we denote by $\overrightarrow{\boldsymbol{R_t}}$ its position about C and $\overrightarrow{\boldsymbol{V_t}}$ its velocity.

Because the system's total momentum in the system's center of mass reference frame is always 0, we have

$$m\overrightarrow{\boldsymbol{v_t}} + m\overrightarrow{\boldsymbol{V_t}} = 0.$$

Thus,

$$\overrightarrow{\boldsymbol{V_t}} = -\overrightarrow{\boldsymbol{v_t}}. \qquad (1)$$

Because there is no external force applied to the system, the system's total kinetic energy is conserved.

The system's initial total kinetic energy is

$$\frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{mv^2}{4}.$$

Hence, the system's total kinetic energy at time t satisfies the following energy conservation equation:

$$\frac{1}{2}mv_t^2 + \frac{1}{2}mV_t^2 = \frac{mv^2}{4}. \qquad (2)$$

Equations (1) and (2) jointly imply $v_t = V_t = \frac{v}{2}$.

Next, we analyze the system's angular momentum. Because there is no external force applied to the system, the system's total angular momentum is conserved. Denote by $\hat{k}$ the unit vector that points towards the inside of the ring.

The system's initial angular momentum about point C is

$$\begin{aligned} m\overrightarrow{\boldsymbol{r_0}} \times \overrightarrow{\boldsymbol{v_0}} + m\overrightarrow{\boldsymbol{R_0}} \times \overrightarrow{\boldsymbol{V_0}} &= m\frac{R}{2}\frac{v}{2}\hat{k} + m\frac{R}{2}\frac{v}{2}\hat{k} \\ &= \frac{mRv}{2}\hat{k}. \end{aligned}$$

We also recall that the ring never rotates about O. Thus, about C, we have the following system's angular momentum conservation equation:

$$m\overrightarrow{\boldsymbol{r_t}} \times \overrightarrow{\boldsymbol{v_t}} + m\overrightarrow{\boldsymbol{R_t}} \times \overrightarrow{\boldsymbol{V_t}} = \frac{mRv}{2}\hat{k}. \qquad (3)$$

We have

$$\begin{aligned} \left| m\overrightarrow{\boldsymbol{r_t}} \times \overrightarrow{\boldsymbol{v_t}} + m\overrightarrow{\boldsymbol{R_t}} \times \overrightarrow{\boldsymbol{V_t}} \right| &= \left| m\left(\overrightarrow{\boldsymbol{R_t}} + \overrightarrow{\boldsymbol{OA_t}}\right) \times \overrightarrow{\boldsymbol{v_t}} + m\overrightarrow{\boldsymbol{R_t}} \times \overrightarrow{\boldsymbol{V_t}} \right| \\ &= \left| m\overrightarrow{\boldsymbol{OA_t}} \times \overrightarrow{\boldsymbol{v_t}} \right| \\ &\leq \frac{mRv}{2}. \end{aligned}$$

The second equality follows from (1). The inequality is binding if and only if both $OA_t = R$ and $\overrightarrow{\boldsymbol{OA_t}} \perp \overrightarrow{\boldsymbol{v_t}}$.

Thus, to satisfy (3), we must have the above inequality binding. That is, we must always have $OA_t = R$ and $\overrightarrow{\boldsymbol{OA_t}} \perp \overrightarrow{\boldsymbol{v_t}}$.

Therefore, all analysis jointly imply that both the point mass A and the center of the ring O are always doing a circular motion about point C in the clockwise direction with the same and constant speed $\frac{v}{2}$. The ring has no rotation about its center O.

Therefore, the time that the point mass A spends to return to its initial position relative to the ring is

$$\frac{2\pi \cdot \frac{R}{2}}{\frac{v}{2}} = \frac{2\pi R}{v}.$$

■

Below is another good example that supports my advice. In the $F = ma$ contest, a large number of problems are circular motions. However, not all problems are exactly the same. In some problems, a point mass makes a circular motion at a constant speed. In some harder problems, the tangential speed may change over time. The most challenging problems even have the radial-component motion.

Although this family of problems have been frequently tested and this type of motions seems quite classical, it does not entail that such problems are always trivial. For example, the acceleration may have its radial-component and tangential-component. However, these are not trivial. There are lots of fundamental questions that need to be clearly figured out. Let me list some (but not exhaustive) below.

1. When is the radial-component acceleration purely a centripetal acceleration?
2. How does the change of the radius affect radial-component acceleration?
3. Under what condition there is non-zero tangential-component acceleration?
4. Is the tangential-component acceleration only generated by the change of the angular velocity?

If you just remember the answers to these special case questions without understanding how they are derived and how they are fundamentally connected with each other, you will have no idea of the answer when you see a new problem that may even just a slight modification of those that you have ever seen.

To fundamentally understand this family of problems and thus correctly solve them, you need to derive general formulas of the velocity and acceleration for the most general setting. In this process, you can have a crystal clear understanding of where each type of acceleration comes from.

To guide you how to derive formulas from scratch, I state the theorem and then follow up with my sample proof. When you prepare for the $F = ma$ contest, you should do the same or similar analysis as I illustrate below (as well as some examples that I illustrate above).

Theorem 1. *A motion in a polar coordinate system is given by $r_t \widehat{\boldsymbol{e_t}}$. Then the velocity at time t is*

$$\overrightarrow{\boldsymbol{v_t}} = r_t \dot{\theta}_t \ \widehat{\boldsymbol{e_\theta}} + \dot{r}_t \ \widehat{\boldsymbol{e_r}}.$$

The acceleration at time t is

$$\overrightarrow{\boldsymbol{a_t}} = \left(\ddot{r}_t - r_t \dot{\theta}_t^{\ 2} \right) \widehat{\boldsymbol{e_r}} + \left(2 \dot{r}_t \dot{\theta}_t + r_t \ddot{\theta}_t \right) \widehat{\boldsymbol{e_\theta}}.$$

Proof. First, we compute $\frac{d\widehat{e_r}}{dt}$. We have

$$\begin{aligned} \frac{d\widehat{\boldsymbol{e_r}}}{dt} &= \frac{d\left(\cos\theta_t \ \hat{\boldsymbol{\imath}} + \sin\theta_t \ \hat{\boldsymbol{\jmath}} \right)}{dt} \\ &= -\sin\theta_t \cdot \dot{\theta}_t \ \hat{\boldsymbol{\imath}} + \cos\theta_t \cdot \dot{\theta}_t \ \hat{\boldsymbol{\jmath}} \\ &= \dot{\theta}_t \ \widehat{\boldsymbol{e_\theta}}. \end{aligned}$$

Second, we compute $\frac{d\widehat{e_\theta}}{dt}$. We have

$$\begin{aligned} \frac{d\widehat{\boldsymbol{e_\theta}}}{dt} &= \frac{d\left(-\sin\theta_t \ \hat{\boldsymbol{\imath}} + \cos\theta_t \ \hat{\boldsymbol{\jmath}} \right)}{dt} \\ &= -\cos\theta_t \cdot \dot{\theta}_t \ \hat{\boldsymbol{\imath}} - \sin\theta_t \cdot \dot{\theta}_t \ \hat{\boldsymbol{\jmath}} \\ &= -\dot{\theta}_t \ \widehat{\boldsymbol{e_r}}. \end{aligned}$$

Now, we compute velocity $\overrightarrow{\boldsymbol{v_t}}$. We have

$$\begin{aligned} \overrightarrow{\boldsymbol{v_t}} &= \frac{d\overrightarrow{r_t}}{dt} \\ &= \frac{d\left(r_t \ \widehat{\boldsymbol{e_r}} \right)}{dt} \\ &= r_t \frac{d\widehat{\boldsymbol{e_r}}}{dt} + \frac{dr_t}{dt} \ \widehat{\boldsymbol{e_r}} \end{aligned}$$

$$= r_t \dot{\theta}_t \ \widehat{\boldsymbol{e_\theta}} + \dot{r}_t \ \widehat{\boldsymbol{e_r}}.$$

Next, we compute acceleration $\overrightarrow{\boldsymbol{a_t}}$. We have

$$\begin{aligned}
\overrightarrow{\boldsymbol{a_t}} &= \frac{d\overrightarrow{\boldsymbol{a_t}}}{dt} \\
&= \frac{d}{dt}\left(r_t \dot{\theta}_t \ \widehat{\boldsymbol{e_\theta}} + \dot{r}_t \ \widehat{\boldsymbol{e_r}}\right) \\
&= \dot{r}_t \dot{\theta}_t \ \widehat{\boldsymbol{e_\theta}} + r_t \ddot{\theta}_t \ \widehat{\boldsymbol{e_\theta}} + r_t \dot{\theta}_t \frac{d\widehat{\boldsymbol{e_\theta}}}{dt} + \ddot{r}_t \ \widehat{\boldsymbol{e_r}} + \dot{r}_t \frac{d\widehat{\boldsymbol{e_r}}}{dt} \\
&= \dot{r}_t \dot{\theta}_t \ \widehat{\boldsymbol{e_\theta}} + r_t \ddot{\theta}_t \ \widehat{\boldsymbol{e_\theta}} + r_t \dot{\theta}_t \left(-\dot{\theta}_t \ \widehat{\boldsymbol{e_r}}\right) + \ddot{r}_t \ \widehat{\boldsymbol{e_r}} + \dot{r}_t \dot{\theta}_t \ \widehat{\boldsymbol{e_\theta}} \\
&= \left(\ddot{r}_t - r_t \dot{\theta}_t^{\,2}\right) \widehat{\boldsymbol{e_r}} + \left(2\dot{r}_t \dot{\theta}_t + r_t \ddot{\theta}_t\right) \widehat{\boldsymbol{e_\theta}}.
\end{aligned}$$

■

4. Advice 3: Have clear physics diagrams

My previous advices are about the necessity and the value of having solid math backgrounds for the $F = ma$ contest. However, physics is not purely an application or a branch of math. The prerequisite of correctly using math is to have clear physics diagrams. Math will then be applied to characterize the physics diagrams.

I have seen some students who are confused or careless about the physics diagrams. They quickly wrote down a math equation without carefully thinking about the physics meaning of each term in the equation. If a physics diagram is wrong, then all subsequent math analysis are meaningless.

Let us look at the following real $F = ma$ problem.

Example 12. *(2023 $F = ma$ Problem 12) A block of mass m is initially held in place by two massless strings, as shown.*

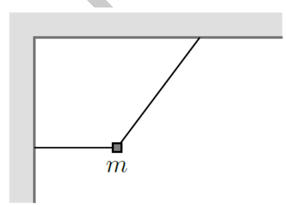

Figure 8: 2023 $F = ma$ Problem 12

The tension in the diagonal string is T_1. Next, the horizontal string is cut, and immediately afterward the tension in the diagonal string is T_2. Which of the following is true?

A. $T_1 < mg < T_2$
B. $T_2 < mg < T_1$
C. $T_1 < T_2 < mg$
D. $mg < T_2 < T_1$
E. $T_1 = T_2 < mg$

In this problem, the physics diagrams before and after the horizonal string is cut are totally different. Before the cut, the physics diagram is a static system. To mathematically formalize this physics diagram, we should write down a math equation that the net force is zero.

By contrast, after the cut, although it seems the system has a minor change, the physics diagram is completely different. The correct physics diagram is that the point mass does a circular motion. To mathematically formalize this physics diagram, we should write down a mathematical equation about the centripetal acceleration.

Let us look at one more real $F = ma$ problem.

Example 13. *(2023 $F = ma$ Problem 9) A helium balloon is released from the floor in a room at rest, then slowly rises and comes to rest touching the ceiling. During this process, the gravitational potential energy of the balloon has increased. Since energy is conserved, the energy of something else must have decreased during this process. Which of the following is the main contribution to this decrease?*

A. The kinetic energy of the balloon decreased.
B. The elastic potential energy of the balloon decreased.
C. The thermal energy of the air in the balloon decreased.
D. The thermal energy of the air in the room decreased.
E. The gravitational potential energy of the air in the room decreased.

This problem does not even require any computational analysis. So it purely tests whether students have a clear physics diagram. The essence of the physics diagram is, when the balloon slowly moves from one position (say, A) to another position (say. B), it essentially exchanges its position with the air who originally occupies position B and then moves to A. Because the system is always static, the gravitational potential energy changes of the balloon and the air that the balloon interchanges its position with offset against each other.

Next, let us look at one more real physics contest problem (from PUPC) about the buoyant force.

Example 14. *(2021 PUPC Problem 1) A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in figure. The tank is filled with water up to a height 0.5 m. The specific gravity of the plank (or ratio of plank density to water density) is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position (exclude the case $\theta = 0$)*

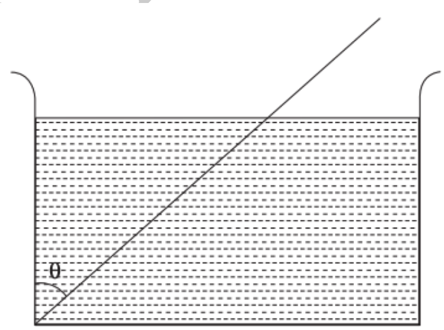

Figure 9: 2021 PUPC Problem 1

To answer this question, a student needs to have a clear physics diagram of the essence of the buoyant force. When students learn buoyant force, they cannot just simply memorize the mathematical formula of the buoyant force without understanding its physics diagram. They should understand the essence of the buoyant force in physics.

The fundament physics diagram is as follows. First, among different objects, as long as their submerged parts have the same shape and are in the same position, the buoyant forces that the surrounding liquid/air apply to them are the same. This is irrespective of the material in the object and the density distribution within the object. Second, to quantify the buoyant force, we consider a special submerged part whose material is exactly the same as the surrounding liquid/air (I hereafter call this "environmentally-homogeneous" submerged part). Because this submerged part is in the static equilibrium, its buoyant force offsets against its gravitational force (i.e., the same magnitude but the opposite direction of the gravitational force, and acting at the center of mass of the "environmentally-homogeneous" submerged part). Third, we can conclude that the buoyant force above applies to any submerged part that has the same shape and position of the above "environmentally-homogeneous" submerged part.

By having this correct physic diagram, for the above PUPC problem, we can correctly and easily determine all elements of the buoyant force, including its magnitude, direction, point that it applies at.

5. Advice 4: Develop a sense of order of approximation

Some students only like exact solutions. They do not like approximations. They incorrectly believe that making approximations is a sign of their incapability of making rigorous analysis. However, this view is totally wrong in physics. For complicated physics systems, there is no way (and also no need) to find exact solutions. Different physics quantities have different orders of magnitude and thus contribute differently to the system. Therefore, making approximations is widely adopted by physicists.

Because making orders of approximations is a necessary intuition and skill in studying physics (as well as many other subjects, such as engineering, economics, finance) and students typically need particular training (beyond AP Physics courses) to get such sense and skill, such problems have been frequently tested in $F = ma$ to effectively distinguish students.

Let me make it more clear for the $F = ma$ contest. There are quite a few problems that even explicitly ask students to take approximations. If you resist on making approximations or have no sense which order of approximations should be kept in your equations and what are higher order terms that should be thrown away, there is no way for you to solve these approximation problems.

One canonical problem is the circular motion of a vertical pendulum with a small angle. If the angle is not necessarily small, the pendulum does not make a harmonic motion. However, if the angle is small, then the approximation analysis shows that the pendulum does a harmonic motion. That being said, the harmonic motion is valid and its mathematical equation is established only if we make approximations with the small angle.

We have also illustrated quite a few $F = ma$ problems in the previous sections of this note that support my advice of developing physics intuition and math skill of making approximations, such as Examples 4, 6.

6. Advice 5: Do not forget the quantitative analysis even problems are asked qualitatively

In $F = ma$, a large body of problems ask students to find order relations of some physics quantities. Accordingly, all choices are also designed qualitatively. As an illustration, the following real $F = ma$ problem is in this style.

Example 15. *(2023 $F = ma$ Problem 13) A uniform box with mass m is at rest on a horizontal surface, and the coefficient of static friction between them is μ_s. A force directed at an angle of $85°$ above the horizontal is applied to the center of the box, with a linearly increasing magnitude $F = \beta t$.*

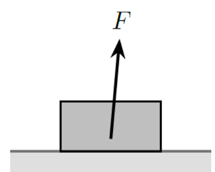

Figure 10: 2023 $F = ma$ Problem 13

The box will eventually slide or lift off the ground. Which of the following is correct?
A. *If $\mu_s < \tan 85°$, the box will lift off the ground first.*
B. *If $\mu_s < \tan 85°$, the box will slide first.*
C. *For any value of μ_s, the box will lift off the ground first.*
D. *For any value of μ_s, the box will slide first.*
E. *The answer depends on the values of β, g, and m.*

Many other $F = ma$ problems that we have illustrated in previous sections are also in this style, such as Examples 2, 9.

This way of designing problems and choices is very misleading for students to find right ways to solve them. Although these problems do not directly and explicitly ask students to do quantitative analysis, a safe, rigorous, and efficient way to answer these qualitative problems is to treat them as quantitative problems and then solve them with quantitative analysis. After getting quantitative results, we then go back to check which choice gives us the correct qualitative description of the quantitative results. For example, in Example 15, we simply compute two threshold times: the time that the box slides (assuming it does not lift off the ground), and the time that the box lifts off the ground (assuming it does not slide). We them compare which threshold time is smaller.

The reason why I use the word "misleading" for such problems is because some qualitative relationships of physics quantities are very deep and cannot be observed from superficial and qualitative intuition. The above problem that I illustrate is a very good example. I have seen quite a few students who got wrong answers to this question. This is a surprise to me. This problem is not hard at all. How can it be true that the mistake rate is so high? I have talked to some students who got incorrect answers. I tried to learn how they solved this problem during the exam. They told me that since the problem was asked in a qualitative way, they incorrectly felt that they could get the correct answer by doing some qualitative comparison between the horizontal move (slide) and the vertical move (lift off). They used the direct but incorrect sense that a problem that asks qualitative questions should be solved in a qualitative way. However, when I asked them to resolve this problem by deriving closed-form formulas of those two threshold times, most of them got the correct answer very quickly.

7. Advice 6: Use symbolic variables throughout the analysis and plug numbers in the very end

Some $F = ma$ contest problems are with numbers. A vast number of students have habits of playing with numbers throughout the whole process of solving a problem, such as writing down equations with numbers, computing with numbers in the intermediate steps. However, this is not a good habit. Reasons are as follows. First, if equations are with lots of numbers and particularly these numbers are not simple and elegant (e.g., very high or low orders of magnitude, many decimals), you are likely to be overwhelmed by numbers, not physics. You may lose your focus on physics itself. Second, making numerical simplifications in the intermediate steps may not lead to a lighten computational burden to get the final answer. Sometimes, the most efficient way to get the final answer is to leave the number manipulation to the last step.

For example, in Example 6, although the problem provides specific numbers, I only plug them in the very end. If you cannot see the value of my delayed numerical computation, I suggest you to simply mimic my solution, except that you plug numbers in the beginning and play with numbers in all steps. As you can see, the numbers given in this problem are at the magnitude of $\frac{\pi}{3600 \cdot 180}$ radian measure. These are very small numbers. If we plug these numbers too early, we have to allocate non-trivial mental concentration on playing with these numbers. This could significantly slow down our analysis and even cause some computational mistake.

8. Concluding Remarks

In this note, I provide a list of advices for students who are preparing for the $F = ma$ physics competition. These advices are from my observation of some common misconceptions that many students had in the past. I do not want to see more students to waste their times to prepare for $F = ma$ on their incorrect paths without even realizing the issue.

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