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Syllabus for the Math+CS Integrated Olympiad Course:

Combinatorics and Probability – Basic Level[1]

PROFESSOR CHEN EDUCATION PALACE

OBJECTIVES

We expect students to achieve the following objectives after taking this course:

1. Build a solid foundation in combinatorics and probability for taking math and CS competitions. This prepares students to
   (1) solve most combinatorics and probability problems in AMC 8 and MATHCOUNTS;
   (2) solve simple and intermediate level combinatorics and probability problems in AMC 10 and 12;
   (3) solve simple and intermediate level combinatorics and probability problems in AIME;
   (4) understand basic combinatorics and probability concepts needed in USACO;
   (5) master basic algorithms with combinatorics and probability tested in USACO;
   (6) develop basic skills on solving combinatorics and probability problems in USACO.
2. Be ready to take our advanced level course in combinatorics and probability. This advanced level course equips students to solve all combinatorics and probability problems in AMC 10, AMC 12, and AIME, and further be prepared for taking USACO.
3. Prepare students to take college-level courses in math and CS, such as discrete math, machine learning, algorithms and data structures.

4. Prepare students to do cutting-edge research that requires knowledge in combinatorics and probability, such as combinatorial optimization, machine learning.

PREREQUISITES

Before taking this course, students are required to know middle-school level math, such as solving a linear equation (example: $23x - 17 = 351$), doing multiplication of polynomials (example: $(x-3)(x^2 - x + 5)$).

Students are NOT required to have any other math or CS backgrounds, such as knowing combinatorics, number theory, trigonometry, or mastering any computer programming language.

DURATION

This is a 20-hour course.

SAMPLE LECTURE

URL: https://us02web.zoom.us/rec/share/uKIqvLWQmLuN0Hi-ob5BBS39c0aChJvfIaDoUVe5GP3m9j2xv9XJN4szY4uuH9FC.o9ZF5WpaIbg9Q6F0

Passcode: 9Xqm.14@

TEXTBOOKS

1. AOPS book: Introduction to Counting and Probability.
2. Our developed course packet.

COURSE CONTENT

Chapter 1: Basic counting techniques

1. Permutations

- Definition of permutation
- Denotation of permutation
- How to compute permutations
- Practice

2. Casework analysis

- Approach of casework
    - make cases according to the characteristics of the problem
    - make cases by supposing possible values of a variable
    - make cases by counting a particular size
- Casework with subcases
- Practice & Challenge problems

3. Complementary counting

- Approach of complementary counting
- Comparison of complementary counting and casework
- Constructive counting
- Practice & Challenge problems

4. Correcting for overcounting

- Permutations with repeated elements
- Counting pairs of items
- Counting with symmetries
- Practice & Challenge problems

Chapter 2: Combinations – Part 1

1. Basics of combinations

- Committee forming: an idea of combinations
- Definition of combination
- Denotation of combination

- How to compute combinations
- Practice

2. Our first combinatorial identity

- The symmetric identity of combinations
- Practice & Challenge problems

3. Paths on a grid

- The model of paths on a grid
- Application of combinations
- Practice

4. Distinguishability

- Boxes and balls: an idea of distinguishability
- Practice & Challenge problems

Chapter 3: Probabilities

1. Basics of probabilities

- Definition of probabilities
- Denotation of probabilities
- Formula of probabilities
- Practice & Challenge problems

2. Classical probabilities

- Probabilities and addition
- Complementary probabilities
- Probabilities and multiplication
- Probabilities with dependent events
- Practice & Challenge problems

3. Geometric probabilities

- Probability using length
- Probability using areas
- Practice & Challenge problems

4. Expectation

- Definition of expected value

- Practice & Challenge problems

Chapter 4: Combinations – Part 2

1. Pascal's triangle

- Constructing pascal's triangle

- Combinatorial identities of pascal's triangle

- Practice & Challenge problems

2. The Hockey Stick identity

- The problem

- The solution

- An combinatorial identity: The Hockey Stick identity

- Practice & Challenge problems

3. The Binomial Theorem

- The Binomial Theorem

- Applications of the Binomial Theorem

- Practice & Challenge problems

Chapter 5: Operations of sets

1. Sets of logic

- Definition of sets

- Denotation of sets and related signals

- Logics and sets

- Practice

2. Principle of inclusion and exclusion

- Principle of inclusion and exclusion

- Applications of principle of inclusion and exclusion

- Practice

Chapter 6: Combinatorics and algorithms

1. Combinatorics and Dynamic Programming
2. Expected Value and Dynamic Programming