2023 AMC 10A
Problems and Answer Key *

PROFESSOR CHEN EDUCATION PALACE

1. (2023 AMC 10A Problem 1)(2023 AMC 12A Problem 1)

   Cities A and B are 45 miles apart. Alicia lives in A and Beth lives in B. Alicia bikes towards B at 18 miles per hour. Leaving at the same time, Beth bikes toward A at 12 miles per hour. How many miles from City A will they be when they meet?

   A. 20 B. 24 C. 25 D. 26 E. 27

   Solution: **(E) 27**.

2. (2023 AMC 10A Problem 2)(2023 AMC 12A Problem 2)

   The weight of $\frac{1}{3}$ of a large pizza together with $3\frac{1}{2}$ cups of orange slices is the same as the weight of $\frac{3}{4}$ of a large pizza together with $\frac{1}{2}$ cup of orange slices. A cup of orange slices weighs $\frac{1}{4}$ of a pound. What is the weight, in pounds, of a large pizza?

   A. $1\frac{4}{5}$ B. 2 C. $2\frac{2}{5}$ D. 4 E. $3\frac{3}{5}$

   Solution: **(A)** $1\frac{4}{5}$.

3. (2023 AMC 10A Problem 3)(2023 AMC 12A Problem 3)

   How many positive perfect squares less than 2023 are divisible by 5?

   A. 8 B. 9 C. 10 D. 11 E. 12

   Solution: **(A) 8**.

4. (2023 AMC 10A Problem 4)

   A quadrilateral has all integer side lengths, a perimeter of 26, and one side of length 4. What is the greatest possible length of one side of this quadrilateral?

   A. 9 B. 10 C. 11 D. 12 E. 13

   Solution: **(D) 12**.

5. (2023 AMC 10A Problem 5)(2023 AMC 12A Problem 4)

   How many digits are in the base-ten representation of $8^5 \cdot 5^{10} \cdot 15^5$?

   A. 14 B. 15 C. 16 D. 17 E. 18

   Solution: **(E) 18**.

6. (2023 AMC 10A Problem 6)

   An integer is assigned to each vertex of a cube. The value of an edge is defined to be the sum of the values of the two vertices it touches, and the value of a face is defined to be the sum of the values of the four edges surrounding it. The value of the cube is defined as the sum of the values of its six faces. Suppose the sum of the integers assigned to the vertices is 21. What is the value of the cube?

   A. 42 B. 63 C. 84 D. 126 E. 252

   Solution: **(D) 126**.

7. (2023 AMC 10A Problem 7)(2023 AMC 12A Problem 5)

   Janet rolls a standard 6-sided die 4 times and keeps a running total of the numbers she rolls. What is the probability that at some point, her running total will equal 3?

   A. $\frac{2}{9}$ B. $\frac{49}{216}$ C. $\frac{25}{108}$ D. $\frac{17}{72}$ E. $\frac{13}{54}$

   Solution: **(B)** $\frac{49}{216}$.

8. (2023 AMC 10A Problem 8)

   Barb the baker has developed a new temperature scale for her bakery called the Breadus scale, which is a linear function of the Fahrenheit scale. Bread rises at 110 degrees Fahrenheit, which is 0 degrees on the Breadus scale. Bread is baked at 350 degrees Fahrenheit, which is 100 degrees on the Breadus scale. Bread is done when its internal temperature is 200 degrees Fahrenheit. What is this in degrees on the Breadus scale?

A. 33 B. 34.5 C. 36 D. 37.5 E. 39

Solution: **(D) 37.5**.

9. (2023 AMC 10A Problem 9)(2023 AMC 12A Problem 7)

   A digital display shows the current date as an 8-digit integer consisting of a 4-digit year, followed by a 2-digit month, followed by a 2-digit date within the month. For example, Arbor Day this year is displayed as 20230428. For how many dates in 2023 does each digit appear an even number of times in the 8-digit display for that date?

   A. 5 B. 6 C. 7 D. 8 E. 9

   Solution: **(E) 9**.

10. (2023 AMC 10A Problem 10)(2023 AMC 12A Problem 8)

    Maureen is keeping track of the mean of her quiz scores this semester. If Maureen scores an 11 on the next quiz, her mean will increase by 1. If she scores an 11 on each of the next three quizzes, her mean will increase by 2. What is the mean of her quiz scores currently?

    A. 4 B. 5 C. 6 D. 7 E. 8

    Solution: **(D) 7**.

11. (2023 AMC 10A Problem 11)(2023 AMC 12A Problem 9)

    A square of area 2 is inscribed in a square of area 3, creating four congruent triangles, as shown below. What is the ratio of the shorter leg to the longer leg in the shaded right triangle?

    A. $\frac{1}{5}$ B. $\frac{1}{4}$ C. $2-\sqrt{3}$ D. $\sqrt{3}-\sqrt{2}$ E. $\sqrt{2}-1$

    Solution: **(C)** $2-\sqrt{3}$.

12. (2023 AMC 10A Problem 12)

    How many three-digit positive integers N satisfy the following properties?

    - The number N is divisible by 7.
    - The number formed by reversing the digits of N is divisible by 5.

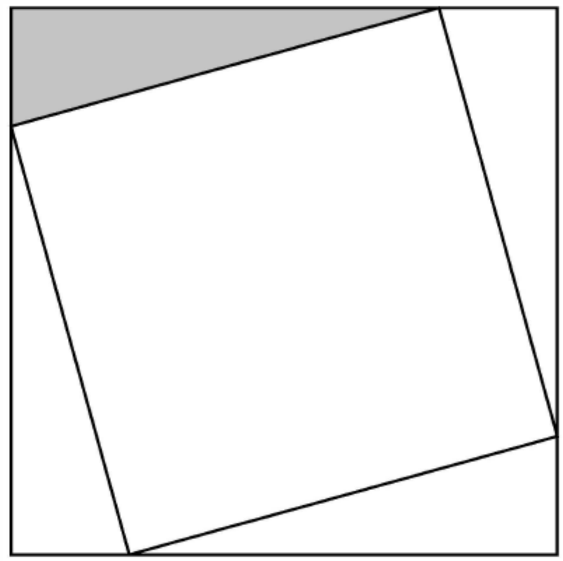

Figure 1: 2023 AMC 12A Problem 9

A. 13 B. 14 C. 15 D. 16 E. 17

Solution: **(B) 14**.

13. (2023 AMC 10A Problem 13)

Abdul and Chiang are standing 48 feet apart in a field. Bharat is standing in the same field as far from Abdul as possible so that the angle formed by his lines of sight to Abdul and Chiang measures $60°$. What is the square of the distance (in feet) between Abdul and Bharat?

A. 1728 B. 2601 C. 3072 D. 4608 E. 6912

Solution: **(C) 3072**.

14. (2023 AMC 10A Problem 14)

A number is chosen at random from among the first 100 positive integers, and a positive integer divisor of that number is then chosen at random. What is the probability that the chosen divisor is divisible by 11?

A. $\frac{4}{100}$ B. $\frac{9}{200}$ C. $\frac{1}{20}$ D. $\frac{11}{200}$ E. $\frac{3}{50}$

Solution: $\boxed{\textbf{(B) } \frac{9}{200}}$.

15. (2023 AMC 10A Problem 15)

An even number of circles are nested, starting with a radius of 1 and increasing by 1 each time, all sharing a common point. The region between every other circle is shaded, starting with the region inside the circle of radius 2 but outside the circle of radius 1. An example showing 8 circles is displayed below. What is the least number of circles needed to make the total shaded area at least 2023π?

Figure 2: 2023 AMC 10A Problem 15

A. 46 B. 48 C. 56 D. 60 E. 64

Solution: **(E) 64**.

16. (2023 AMC 10A Problem 16)(2023 AMC 12A Problem 13)

In a table tennis tournament every participant played every other participant exactly once. Although there were twice as many right-handed players as left-handed players, the number of games won by left-handed players was 40% more than the number of games won by right-handed players. (There were no ties and no ambidextrous players.) What is the total number of games played?

A. 15 B. 36 C. 45 D. 48 E. 66

Solution: **(B) 36** .

17. (2023 AMC 10A Problem 17)

Let $ABCD$ be a rectangle with $AB = 30$ and $BC = 28$. Points P and Q lie on $\overline{BC}$ and $\overline{CD}$, respectively, so that all sides of $\triangle ABP$, $\triangle PCQ$, and $\triangle QDA$ have integer lengths. What is the perimeter of $\triangle APQ$?

A. 84 B. 86 C. 88 D. 90 E. 92

Solution: **(A) 84**.

18. (2023 AMC 10A Problem 18)

A *rhombic dodecahedron* is a convex polyhedron where each of the 12 faces is a rhombus, and all of the faces are congruent to each other. The number of edges that meet at a vertex is either 3 or 4, depending on the vertex. What is the number of vertices at which exactly 3 edges meet?

A. 5 B. 6 C. 7 D. 8 E. 9

Solution: **(D) 8**.

19. (2023 AMC 10A Problem 19)

The line segment from $A(1,2)$ to $B(3,3)$ can be transformed to the line segment from $A'(3,1)$ to $B'(4,3)$, sending A to A' and B to B', by a rotation centered at the point$P(s,t)$. What is $|s-t|$?

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{2}{3}$ D. $\frac{3}{4}$ E. 1

Solution: **(E) 1**.

20. (2023 AMC 10A Problem 20)

Each square in a 3×3 grid of squares is colored red, white, blue, or green so that every 2×2 square contains on square of each color. One such coloring is shown on the right below. How many different colorings are possible.

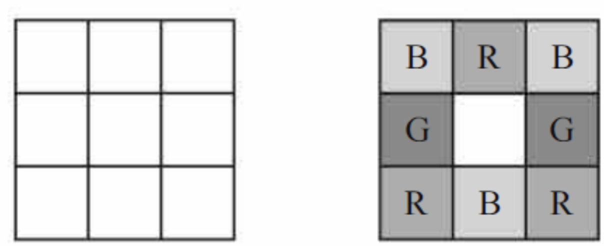

Figure 3: 2023 AMC 10A Problem 20

A. 24 B. 48 C. 60 D. 72 E. 96

Solution: **(D) 72**.

21. (2023 AMC 10A Problem 21)

Let $P(x)$ be the unique polynomial of minimal degree with the following properties:

- $P(x)$ has leading coefficient 1,
- 1 is a root of $P(x) - 1$.
- 2 is a root of $P(x-2)$.
- 3 is a root of $P(3x)$.
- 4 is a root of $4P(x)$.

The roots of $P(x)$ are integers, with one exception. The root that is not an integer can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

A. 41 B. 43 C. 45 D. 47 E. 49

Solution: **(D) 47**.

22. (2023 AMC 10A Problem 22)(2023 AMC 12A Problem 18)

Circles C_1 and C_2 each have radius 1, and the distance between their centers is $\frac{1}{2}$. Circle C_3 is the largest circle internally tangent to both C_1 and C_2. Circle C_4 is internally tangent to both C_1 and C_2 and externally tangent to C_3. What is the radius of C_4?

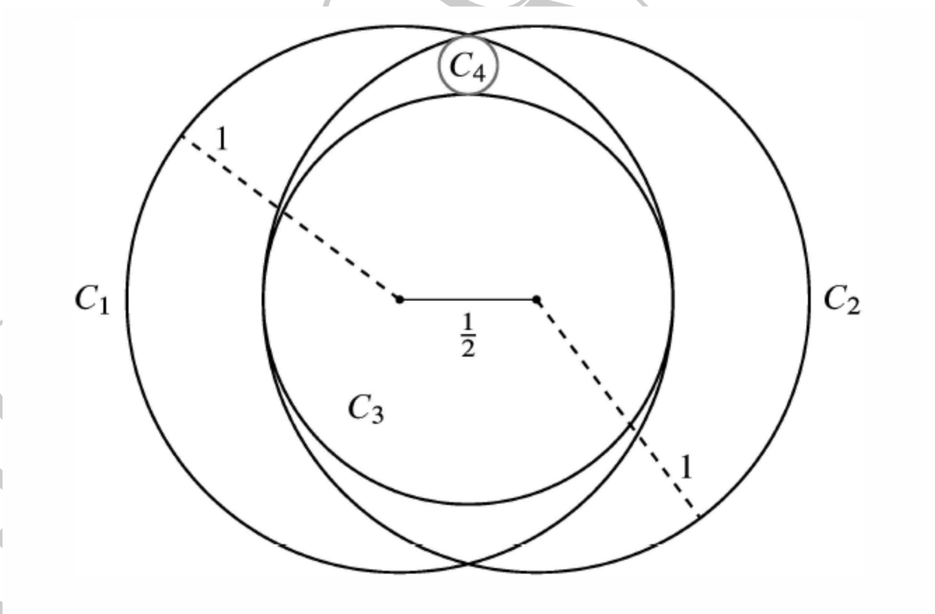

Figure 4: 2023 AMC 12A Problem 18

A. $\frac{1}{14}$ B. $\frac{1}{12}$ C. $\frac{1}{10}$ D. $\frac{3}{28}$ E. $\frac{1}{9}$

Solution: $\boxed{\textbf{(D)}\ \frac{3}{28}}$.

23. (2023 AMC 10A Problem 23)

If the positive integer c has positive integer divisors a and b with $c = ab$, then a and b are said to be *complementary* divisors of c. Suppose that N is a positive integer that has one complementary pair of divisors that differ by 20 and another pair of complementary divisors that differ by 23. What is the sum of the digits of N?

A. 9 B. 13 C. 15 D. 17 E. 19

Solution: $\boxed{\textbf{(C)}\ \textbf{15}}$.

24. (2023 AMC 10A Problem 24)

Six regular hexagonal blocks of side length1 unit are arranged inside a regular hexagonal frame. Each block lies along an inside edge of the frame and is aligned with two other blocks, as shown in the figure below. The distance from any corner of the frame to the nearest vertex of a block is $\frac{3}{7}$ unit. What is the area of the region inside the frame not occupied by the blocks?

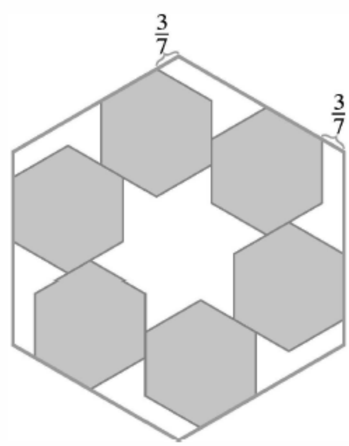

Figure 5: 2023 AMC 10A Problem 24

A. $\frac{13\sqrt{3}}{3}$ B. $\frac{216\sqrt{3}}{49}$ C. $\frac{9\sqrt{3}}{2}$ D. $\frac{14\sqrt{3}}{3}$ E. $\frac{243\sqrt{3}}{49}$

Solution: $\boxed{\textbf{(C)}\ \frac{9\sqrt{3}}{2}}$.

25. (2023 AMC 10A Problem 25)(2023 AMC 12A Problem 21)

If A and B are vertices of a polyhedron, define the distance $d(A, B)$ to be the minimum number of edges of the polyhedron one must traverse in order to connect A and B. For example, if $\overline{AB}$ is an edge of the polyhedron, then $d(A, B) = 1$, but if $\overline{AC}$ and $\overline{CB}$ are edges and $\overline{AB}$ is not an edge, then $d(A, B) = 2$. Let Q, R, and S be randomly chosen distinct vertices of a regular icosahedron (regular polyhedron made up of 20 equilateral triangles). What is the probability that $d(Q, R) > d(R, S)$?

A. $\frac{7}{22}$ B. $\frac{1}{3}$ C. $\frac{3}{8}$ D. $\frac{5}{12}$ E. $\frac{1}{2}$

Solution: $\boxed{\textbf{(A)}\ \frac{7}{22}}$.